RETAILER DECISION-MAKING: THE PRICE OF SUPPLIES

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This paper presents a new approach to determining the price that consumers pay for goods and services, hereafter referred to as the price of supplies. Unlike the various current theories of supply and demand, which depend upon concepts that cannot be validated against observed data, the new approach represents verifiable mechanisms in the pricing of supplies, which will produce repeatable predictions. This means that it can be validated against observed features of an economy, such as the change in inflation or tax revenues over time.

In the new theory, the factors that affect the availability and cost of business input supplies, which are produced in the past and in different economic regions, are accounted for in determining the real cost price of production, which can be calculated accurately. The retail price is the cost price plus a profit margin. As the retail price is constrained by business-to-business competition and consumer behaviour, a probabilistic approach based on historical data is required to calculate profit.

This theory explains why the price of goods and services vary between economies, between regions of an economy, and between producers, and why the price changes over time. Moreover, the cost price of supplies is not only a monetary value, but is also an environmental footprint, such as the volume of greenhouse gases emitted during manufacture and distribution. The methodology presented here, by modelling the cost price of production, can calculate the non-monetary cost too.

This paper defines a model that represents the decision-making processes of retailers within the large-scale economic forecasting framework known as Economy Dynamics. The model is presented in two formulations: 1) to reflect the retail price setting decision-making process of individual businesses, which should align with the experience of people involved in these decisions, and 2) implementation of the decision process for all business categories in all economic regions, thereby transforming the first formulation into the Economy Dynamics computational framework.

Access to suitable datasets will be a constraint for the validation of the model as existing data is collected in a form aligned with current economic theories. This means that it cannot be used for validation as it must be disaggregated by economic region, and by business and by household economic subgroups. However, given financial support for the project then appropriate data can be collated. For now, this paper demonstrates the concepts involved in determining the price of supplies within a model that can be validated.

The importance of validating theories cannot be overstated. For example, as current economic theories cannot be validated, the central banks who control monetary policy, including increasing interest rates, currently do not know whether their actions are beneficial or detrimental to the behaviour of the economy. This is problematic as monetary policies inherently disadvantage parts of the economy whilst benefitting others. For example, increasing interest rates directly disadvantages those who need to borrow money and directly benefits those who have money to save and invest. Validating economic theories will separate economics from politics, so that policies can be explained and justified to the public. This will lead to genuine political choice about the ethics and environmental sustainability of economic policies.

1 INTRODUCTION

1.1 *THE PROBLEM OF CURRENT THEORIES*

The current approach to economic theory is divided into macroeconomics, the study of markets and economies, and microeconomics, the behavioural study of businesses and households. These are thought to be two distinct academic fields that do not interrelate, similar in nature to the apparent discontinuity between theories of classical (large scale) and quantum (small scale) physics. This means that theories of large-scale economic behaviour are believed to function through processes that cannot be observed at the small-scale. For example, it is thought that inflation cannot be explained by the financial interactions of businesses.

In fact, the theory of supply and demand neglects a crucial observable factor. Namely, that retailers set the price without knowledge of real-time supply and demand. So, the problem with current theories such as Arrow and Debreu (1954) and Samuelson and Nordhaus (2009) is that they cannot explain how prices are determined. A person on behalf of the business must change the price, but they only know the historical volume of supplies produced and sold by the business they work for. They cannot know the real-time economy level information such as total supply of all competing producers and total demand from all consumers.

To determine how retailers set the price, we could observe the decision-making processes of businesses, but current large-scale economic theories reject these studies owing to the belief that they are not relevant. The result is that although concepts may appear intuitive, such as price being determined by the level of consumer demand, this is not sufficient for a general explanation of economy behaviour.

The foundation of economic theory claims that supply and demand determine price, but there are no practical models that clearly prove this against observable data. Therefore, this paper presents a mathematical exposition of the observable and verifiable processes by which businesses set the retail price including the data requirements for this decision-making. The predictions of the model can be validated, which means that its forecasts can be plotted by independent researchers against any appropriate empirical dataset.

1.2 *THE NEED FOR THIS PAPER*

All scientific theories start as a guess, as discussed by Feynman (1964). The only way to know if the guess represents fiction or reality is to validate the theory against observation. This means that the quality of the theory's predictions can be compared to empirical data to test how well it explains observed behaviour. Crucially, validation enables us to prove if one theory is better than another. Selecting the best theory provides us with useful information as demonstrated by engineering and medicine.

If a theory that supposedly defines real world processes cannot be validated, it cannot be considered as scientific. Rejecting theories that cannot be validated ensures that knowledge advances by building new testable hypotheses upon proven knowledge. In contrast, economics is a subject that does not require theories to be validated, as demonstrated by the concept of supply and demand, such as Equilibrium Theory proposed by Arrow and Debreu (1954), and the ideas taught to undergraduates (such as, Samuelson and Nordhaus, 2009).

Problematically, economic policy is currently based on unvalidated theories. For example, as there are no validated theories of inflation, no one understands the cause of inflation. Yet the central banks determine base interest rates by using unvalidated Quantity Theory, which postulates that the total amount of money available to all consumers in an economy affects the price of supplies. So, policymakers believe that too much money in the hands of consumers causes inflation. The result is that when inflation rises above a threshold based on their judgement, the central banks increase base interest rates to reduce the money available to consumers.

Implementing any policy based on unvalidated theories will have unintended consequences, as the policymaker cannot know the potential detrimental effects of the theory. Indeed, economic policies inherently disadvantage parts of the economy whilst benefitting others, such as interest rate rises increasing the cost of borrowing for some households and businesses, whilst producing greater returns on savings for other households. Moreover, it is probable that many of the current global issues, such as the cost-of-living crisis, are unintended consequences of the current economic system and economic policies.

Economy Dynamics is being developed as an economic forecasting framework. This paper presents a module for modelling the price of supplies that can be validated within the forecasting framework against observed features of the economy.

1.3 THE NEED FOR VALIDATING THEORIES

1.3.1 NEGLECTING INFORMATION HAS CONSEQUENCES

The approach to current economic theory is based on logical reasoning. The philosophy of this approach is more precisely known as deductive reasoning, whereby the validity of a conclusion is dependent on the validity of a logical premise. The standard example of deductive reasoning is as follows: all men are mortal; Socrates was a man; therefore, Socrates was mortal. The problem with applying deductive reasoning to a complex system such as an economy is its inability to deal with multiple inter-related variables. Therefore, the validity of the logical premise is dependent on the specific economic situation.

In economic theory, for example, to argue that the value of labour is in the value of the goods has a basic element of truth. The wage a business can pay is constrained by the price customers pay for the goods sold and the effort it takes to produce those goods. The logically deduced statement sounds plausible making it difficult to reject. Stating that the value of labour is in the value of goods is selectively extracting one mechanism that affects wages and exaggerating its importance. It feels intuitive, but is misleading, as it does not consider the wider context of economy behaviour.

The problem with logical deduction based on pure reasoning is that theories cannot handle complex relationships between different variables within an economy. This results in a grossly simplified version of the situation that does not represent reality. For example, businesses have many different job roles and not a single wage category; there is variation in regional cost-of-living which can affect the ability to live on a wage; and competition between businesses in different business sectors and economic regions to recruit staff means wages cannot be determined by a single business. The rate for wages will be defined in a subsequent Economy Dynamics model (Maybury, in progress, [Paper 07]).

The use of deductive reasoning in current economic theories leads to the need for idealised assumptions, which deliberately neglect the information that deductive reasoning cannot handle. When a theory that is missing information is applied to a real-world scenario it will have unintended consequences. This is discussed in detail by Maybury, 2022 [Book 01, Preprint B1-1-5].

1.3.2 Representing the complete system

The current approach to economic theories neglects information but this doesn't need to be the case. The philosophical approach to knowledge used in science and engineering ensures information is not neglected and is applicable to the study of economics. A scientific approach to economics requires all mechanisms to be considered when establishing hypotheses that explain system behaviour. A *mechanism* can be defined as any event or feature that changes the probability of a financial transaction occurring. This includes taxation that affects household income, and business-to-business competition that affects the price of goods.

Unlike current economic forecasts, as the new approach can be validated it provides repeatable, quantifiable errors between its predictions and observed data. These errors provide useful insight into why under certain conditions the model, representing the effects of hypothesised mechanisms, fails to explain empirical trends and patterns. This means that through detailed analysis of the cause of the errors, researchers can establish when and why the mechanisms modelled are not appropriate to forecast a particular phenomenon under certain conditions. This then guides researchers to improve upon existing theories or develop new hypotheses. In this approach, it is possible to ensure that all mechanisms are appropriately represented.

In addition, techniques from engineering enable complex models to be built through piecewise validation of each component, so complexity can be overcome. Despite this, the current unvalidated theories in economics make it impossible to conceptualise how a new theory could account for every mechanism involved in economy behaviour. It is not possible to model a system where all financial transactions are individual choice, as postulated by Utility Theory. However, as Utility Theory is unvalidated, Economy Dynamics hypothesises that most consumer financial transactions are regular repeat purchases or are contractually committed spending (Maybury, 2022 [Book 01, Preprint B1-1-2]). Therefore, the behaviour would be probabilistic and can be derived from population data. To account for probabilistic similarities in financial behaviour of consumer and producer subgroups, the forecasting framework separates the economy into distinct regions, splits consumers into subgroups of similar financial behaviour, disaggregates supplies and assets into subgroups with common features, and divides occupations into job categories.

Naturally, in the fully expanded form, the simulation will be labour intensive to set-up, and computationally intensive to run. However, any part of the input data structure can be aggregated whilst other parts can remain disaggregated (Moffatt, 2019). This means that the simulation fidelity in important aspects of the economy behaviour required for the study can be high whilst less critical aspects of the simulation can be lower. For example, most supplies could be aggregated to the top level (food, clothing, housing, health, transport, …) with the option to represent critical supplies, at a higher level of disaggregation, such as food subdivided into further categories (cereals, vegetables, fruit, meat, fish, …). Further details of the data structure of supplies is provided by Maybury, 2020 [Paper 01]. This approach enables representation of the whole system, without deliberately neglecting information as any aggregation of data is known and its effects on the forecasts can be studied.

1.4 DEFINITION OF SUPPLIES

All supplies can be sold to both households and businesses. This means that businesses are consumers, as well as households. Yet, current economic theory neglects the consumer behaviour of businesses, and the impact this has on the price of supplies, such as businesses purchasing cars. This is because the expenditure of businesses on input supplies is not equivalent to the spending of household wages on goods and services in the calculation of the size of an economy. For example, Gross Domestic Product (GDP) which measures the size of an economy, when calculated using the National Income method, sums all household wages and all business profits. Therefore, the measure of the size of an economy excludes the spending of businesses on input supplies. So, it may seem logical that households are consumers and businesses are not (or at least are not end consumers). However, this distinction neglects to account for businesses buying the same supplies as households, such as cafes and households buying bread from the same bakery, that has been baked from flour ground by another business from wheat grown by another.

In current economics, due to the separation of household and business consumer behaviour, the term *commodities* is used to define the (interchangeable) input supplies of businesses, which are used to produce other goods and services. This term aligns to the misuse of GDP as a policy-making metric[1] and not to the observation that business and households purchase the same supplies. Therefore, within this paper, commodities are treated as a subset of supplies and not an independent category.

The term *supplies* within this paper refers to the goods and services that constitute the regular and committed costs of households and the input costs of businesses. The supplies for households include housing, food, transport and energy (Maybury, 2020 [Paper 01]). The definition of supplies in this paper is equivalent to the consumer basket of goods and services defined by the Office for National Statistics (ONS.Gov.UK, 2019 [1] and [2]), which is used to calculate UK inflation rates. The input supplies for businesses are those used in manufacturing, sales and marketing, business operations and the distribution of products (in progress, Paper 05). This includes all supplies that can be bought by households. It excludes assets that do not depreciate.

1.5 PAPER OVERVIEW

The methodology defined in this paper is for forecasting the price of supplies as part of the *Economy Dynamics* framework (see Maybury, 2020 [Paper 00]). The structure of this paper is as follows:

Section 1 Introduction. The need for this paper.
Section 2 The price setting system. The observable price setting mechanisms.
Section 3 Price constraints. The mathematics from a business perspective.
Section 4 Resolving the price of supplies. The computational framework.
Section 5 Discussion. Implementation and future validation.
Section 6 References.

[1] The change in size of the economy tells us nothing meaningful about the behaviour of an economy and how to manage it. It only provides information on the wealth generated by all businesses, and not how that wealth is distributed between wages and profits, nor between different regions of an economy, nor between different industrial sectors, and nor between household economic groups. Instead of using GDP, policymakers must monitor the variables of interest in the region, sector, or economic group of interest. Meaningful monitoring variables include the level of unemployment in different regions, and the tax revenues collected by region and sector.

Section 7 Document information.

2 THE PRICE SETTING SYSTEM

The price of goods and services vary between economies, between economic regions, between retailers, and between producers, and the price changes over time. The general characteristics of the system are that:

- Businesses set the retail price.
  - Production costs constrain the minimum retail price.
  - Undesirable business practices have a cost advantage, such as dumping waste in waterways.
  - Business competition constrains the maximum retail price.
  - Profits must meet business needs beyond rewards for financial investors.
- Regions and industrial sectors respond differently to economic stimulus.
- Demand is inherently stable for essential supplies, such as food and drink.
- Retail price affects consumer choice.
- Businesses have incomplete information.

These characteristics are discussed in the sections below. They apply to most markets. The markets controlled by monopolies are not in the scope of this paper.

2.1 THE NATURE OF THE SYSTEM

2.1.1 BUSINESSES SET THE RETAIL PRICE

When businesses set a price for supplies there is a well-defined lower constraint, which is the unit cost price of production. In contrast, the upper constraint, largely driven by competitor pricing, is also affected by unique selling points. As a result, uncertainty over the willingness of consumers to pay higher prices, for say higher quality, more ethical and sustainable brands, means that price setting is a trial-and-error process.

Initially, price may be based on market research of competitor products, although this must be greater than the estimated cost price of production. For new businesses or for new supplies, there is substantial scope for human error in setting the initial price. As the sales history of the supplies mature, then the information from the business's own track record on price and the sales associated with that price will influence future pricing strategies.

2.1.1.1 Cost of production

Businesses consume input goods and services in addition to demanding labour, land, and financial services. For example, a bakery purchases flour, milk, eggs, electricity, and ovens, hires staff, rents property and borrows money. The consumable input costs, staff wages, asset depreciation and interest costs determine the cost price for producing bread.

2.1.1.2 Undesirable business practices have cost advantage

Some unethical and some environmentally unsustainable business practices reduce the costs of production. We can define these terms, that have commercial benefit, as follows:

Unethical Business Practices: The exploitation of people or mistreatment of animals. The exploitation of people ranges from discrimination between people in their rate of pay for work, to the use of child labour, such as in the cocoa industry (Wexler, 2020). In relation to animal welfare, it could be any treatment of domesticated animal that does not align with the designated standards (of a given economy) on animal husbandry.

Unsustainable Business Practices: Generally, these are processes that are performed by businesses to meet the current demand that will hinder the ability of the future generations to meet their needs. More specifically, these processes adversely and irreversibly affect the environment. They come under the categories of the mismanagement of natural resources, such as deforestation and large-scale monocrop farming, and the generation of pollution, such as the emission of greenhouse gases, generating plastic waste, and dumping environmentally damaging waste into waterways. A good example would be overfishing, which reduces the availability of fish to catch in the following year.

2.1.1.3 Price competition

Generally, there is competition between businesses on price, and this imposes an upper constraint on the retail price of supplies. This is determined by the retail price of similar supplies, in a similar market niche (such as the perception of quality) set by the respective business competitors. Where competition is over price alone, then companies must achieve a sales price below that of direct competitors.

Competition between many businesses is more nuanced than simply a price comparison. There are differences in consumer perception of production quality, ethics, sustainability, delivery speed and reliability. For many of these companies there will also be a competition over price, though there may be a price tolerance accepted by the consumer; a value above a competitor product price that the consumers will not choose to switch to the cheaper product. Toothpaste is probably a good example. This is broadly called brand loyalty. To model this, when a company compares the price with similar or alternative goods, they can accept a small increase in price over the competitor.

There are rare companies, that tend to be highly recognisable brand names, who may not need to compare prices to the competitors and may rely on customer loyalty, due to having a more affluent customer base, and a reputation for high production quality, or ethics, or sustainability. This isn’t to say they can set any price, as they could lose consumers on affordability. However, they may be able to set a consistent profit margin without market price comparison.

There are many business models that vary by business sector and business category, such as a small independent bakery or a national airline. This aspect of the model will require substantial research. However, these three categories seem to be a reasonable starting point between competition only over price, such as the value range of supermarkets; companies that participate in nuanced competition, such as breakfast cereal manufacturers; and companies with high brand loyalty, such as household names of the mobile phone sector.

2.1.1.4 The need for profits

The minimum viable retail price is the cost price of the production, plus the minimum profit margin needed to manage cashflow risks, for reinvestment in the business, and for payment to the owners to meet their living costs (see Maybury 2020 [Paper 01]).

Most businesses (~97%) employ less than 10 people and most of these (~75%) the business owner is effectively self-employed (Young, 2012; Rhodes, 2018). The profits of these micro businesses provide income to the business owners in return for their time in lieu of wages. However, unlike wages, a business can only pay dividends if it has profits. It is important that the model appropriately treats profit, where applicable, as wages. As, for some companies, viability is not a cash-flow issue, but a household living-cost matter. If the business owner-workers are unable to pay themselves sufficient dividends to meet their living costs, then the business is not viable.

To gauge the minimum viable profit margin, historical business data can be used to establish a nominal baseline expectation (Paper 05, in progress). This margin will be affected by the business size and maturity, and the business sector it functions within. If the margin is not maintained, based on the profit-and-loss accounting, this will drive business decision-making about pricing.

2.1.2 RESPONSE TO ECONOMIC STIMULUS

2.1.2.1 Regional and temporal effects

The cost of production is affected by the economic circumstances of different regions, industrial sectors and business categories. This is because the goods produced today are made from input supplies purchased in the past. This past cost was in turn affected by the cost of labour and fees for land usage where the input supplies were produced. The effect of the cost of labour and fees for land usage will also vary between different economic regions and business categories.

A simple scenario of a bakery highlights the variables within the cost price of production. The bakery, which produces bread and cakes will buy yeast, flour, sugar, eggs, and milk. These may be purchased on the previous day, such as milk, or on a weekly basis such as flour and sugar, or monthly such as yeast. If the bakery is in a city, then the input goods will have been produced in a different economic region and at a past time. In addition, the wages of staff, within the subgroups of all households, the fees for electricity and water and the commercial rents are costs incurred during the production process.

Each input supply for any business is an output supply from another business produced in the past and, frequently, in a different economic region. The schematic in Figure 1 represents the production of a good in an economic region within Great Britain. Time moves incrementally from left to right, with the timestep denoted at the top of the image. The British economy is divided into regions based on the local economic characteristics for businesses, such as business rates and other taxes. Similarly, the interconnected global economy is also subdivided into distinct economic regions, albeit much larger regions. The schematic represents the continuum of output supplies from one business becoming an input supply to another.

The economic region selected by the business for its production facility, and its general operations, affects its commercial advantage. Similarly, a business's supply chain affects its competitiveness. Time and regional factors mean that the effects of economic events, such as resource depletion, the discovery of new resources, the influence of political unrest or environmental disaster, such as drought, will affect input supply availability and this is reflected in the calculation of business input supply costs. By modelling these dynamically, economic events will ripple through the supply chain and out into co-dependent supply chains, as businesses seek to maintain competitive supply costs.

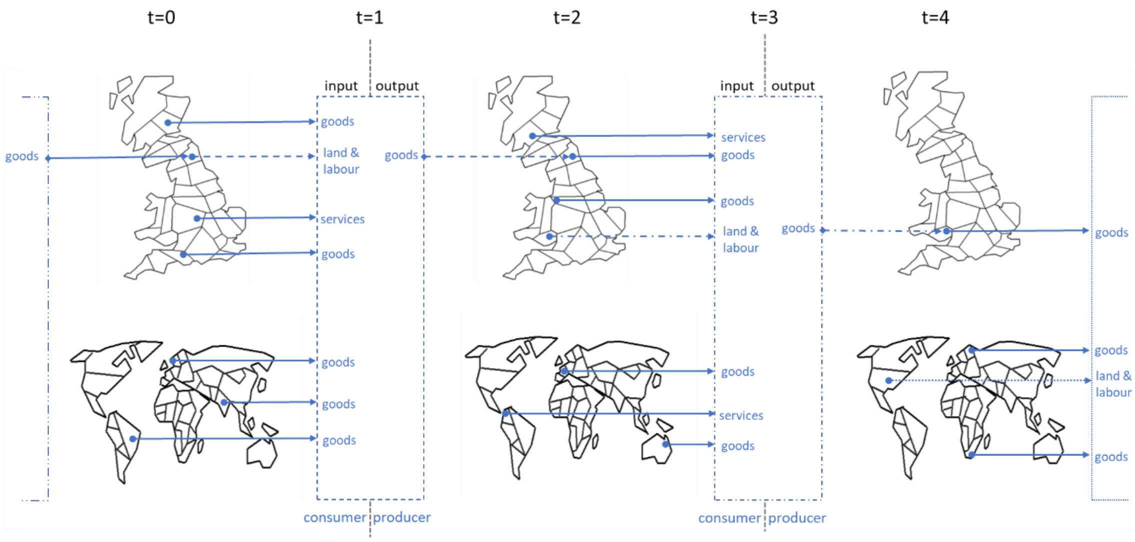

Figure 1. Resolving supply cost price for regional and temporal supply inputs.

2.1.2.2 Business cycle

To model the effects of a supply chain on cost price, we must consider the time dependencies of a business cycle. Observation shows us that businesses tend to fail due to cashflow issues, which is not the same as a lack of profitability. Cashflow refers to the availability of money to pay suppliers, staff, debt repayment, and taxes. It demonstrates the dynamic nature of businesses: namely that input costs tend to be paid before the business has produced the supplies for sale, and receipt of payment is after the sales. A business cycle as three distinct stages: procurement, production, and sales, shown in Figure 2.

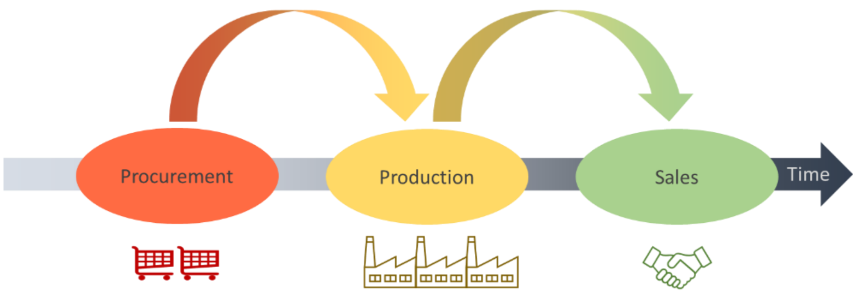

Figure 2. The business cycle schematic: Procurement-Production-Sales

Procurement ensures that the input supplies are available to produce output supplies, such as:

- An arable farmer needs seeds and machinery.
- A car manufacture needs component parts and machinery.
- An accountant needs software, and stationery.

Production is the activity of mining, growing, or manufacturing new supplies, such as:

- A construction firm requires labour and land to build properties.
- A bakery requires labour and land to produce bread.
- An engineering consultancy requires labour and land to deliver services to manufacturers.

Sales convert the output supplies into business income, such as:

- A retail store must stock its shelves before selling the goods.
- A mobile phone must be in its box before it is sold.
- A doctor must examine a patient, or provide treatment, before they submit their bill.

The dynamic nature of cashflow is due to the time delay between procurement, and production, and sales. Figure 3 shows a single business cycle for a hypothetical business; a small artisan bottle producing business. The unit cost of production has been normalised to £100, for a batch of 20 bottles. The simulation timestep is one week, so that the full business cycle is just under three calendar months.

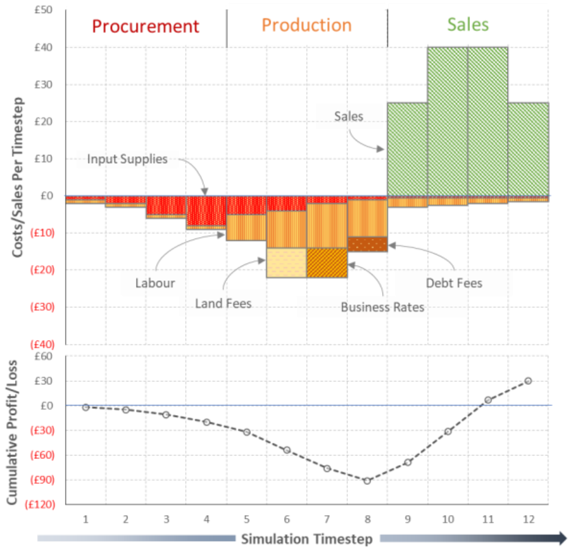

Figure 3. A single business cycle for a £100 unit cost price of production.

For the bottle making business, procurement ensures availability of the raw ingredients, including sand, limestone, soda, and recycled glass. The production process also requires energy and water, and the sales process needs to package and transport the finished articles. The total cost of input supplies is £30 per unit of goods produced.

The manufacturing process is labour intensive, and labour is also needed during the procurement and sales stages. The total price of labour is £50 per unit of goods produced. Other costs are business rates at £8 per unit of goods per month, they lease the land and equipment at a cost of £8 per unit of goods per month and they service debts at £4 per unit of goods per month.

They have an agreement to deliver a given number of units each month to a distillery for which they are paid £130 per unit. This is a 23% profit margin (explained in Section 3.3).

The business operates in a continuous cycle, whereby a single cycle is overlapped by the proceeding cycle and by the following cycle. So, in every calendar month: procurement ensures the input supplies are available for the next manufacturing stage; manufacturing ensures there are goods to sell in the next sales phase; and so, sales occur every month. If we take each stage of the business cycle from Figure 3 and represent it in a continuously repeating pattern, we obtain the snapshot of business activity shown in Figure 4.

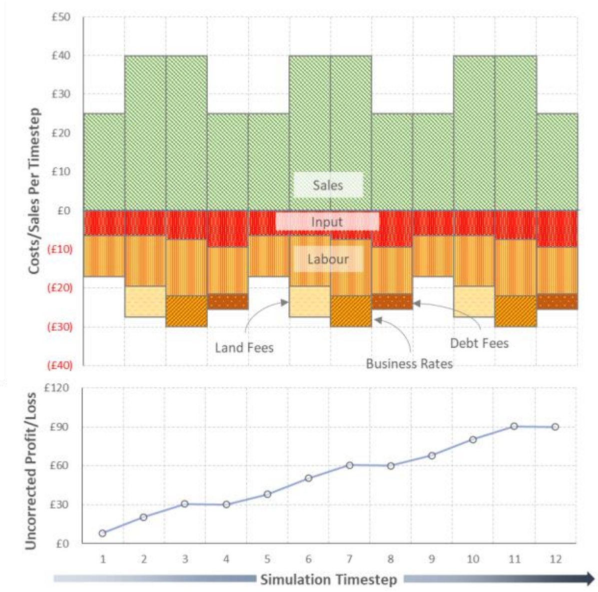

Figure 4. A continuous business cycle for a £100 unit cost price of production.

When the start and end of the business cycles are neglected, it appears that each week the business profit grows, but we know that a business must purchase input supplies before it can manufacture goods. So, how do we reflect this time-lag in economic modelling. Fortunately, this is handled by company annual accounts.

2.1.2.3 Company accounts

Company accounts are divided into two sections: the balance sheet, and the profit & loss account. The balance sheet records the money in the bank and the value of stock, including unused input supplies, goods in production, and goods that have been produced but not sold. The profit & loss account records the operational costs and the value of the sales, but it must also account for stock recorded in the balance sheet.

The balance sheet requires valuing inventory, including raw materials, works in progress, and unsold finished products. There are several methods to calculate the inventory value. The “first in, first out” method will be assumed for convenience, whereby, in the case of a retail business, the first item purchased is the first item sold, and in the case of a manufacturing business, the first item purchased is the first item used in producing an output supply, and these are sold in the order of production.

Crucially, in the company accounts the initial sales of an accounting period are excluded, as those sales are attributed to the stock held on the balance sheet from the previous accounting period, when the input goods were purchased and the product was manufactured. Figure 5 (left) represents the effect of this approach, which creates a loss, as the cost of the procurement and the manufacturing phase in the first four weeks (of the example) are not offset by any sales. In contrast, at the end of the current company accounting period (at week 52) the stock held, which is anticipated to represent sales in the next accounting period, is included. Figure 5 (right) represents the effect of this approach, which creates a rise in profits, as the cost of the procurement and manufacturing has already been accounted for in the preceding weeks.

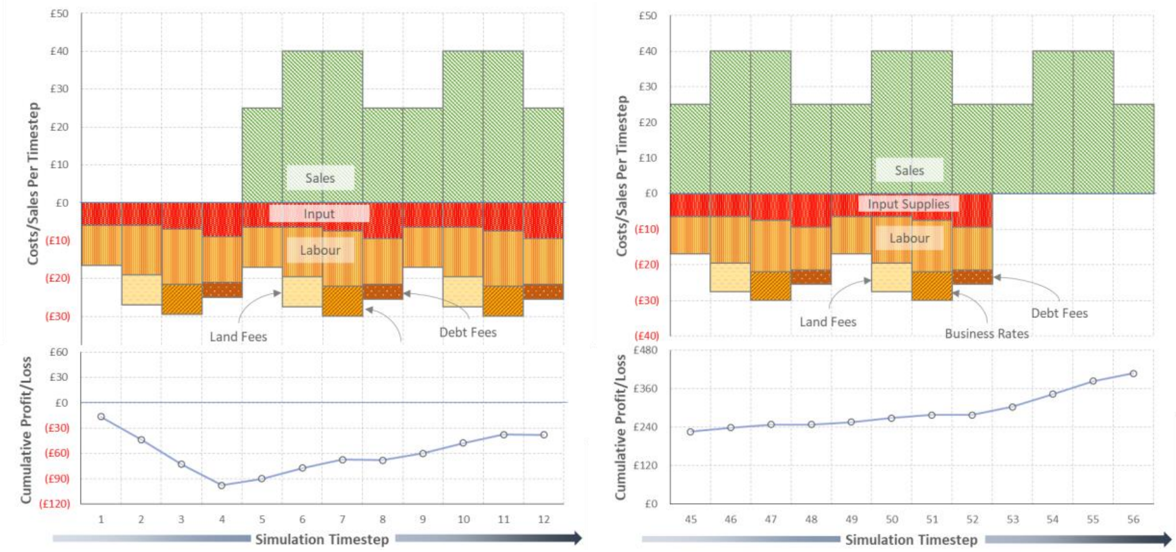

Figure 5. The company accounting period: left) the start and right) the end.

2.1.2.4 Modelling inflation

The representation of the time delay between procurement, manufacturing and sales is vital to model inflation. The effect of a change in the price of input supplies on the cost price of production will have a time delay, by around one complete business cycle. This means that different business categories, such as multi-national or micro enterprises, and different business sectors, such as food production or the automotive industry, are affected by inflation differently, whether it is dominated by input supplies, labour costs or land fees, and they will pass-on the costs over a different time frame to their customers. Therefore, to model inflation and its effects, we must represent the business cycle, whereby procurement proceeds manufacturing which proceeds sales. For example, the automotive industry may take several months to transform input supplies into a product recovering applicable inflationary costs, whereas a bakery may take days.

2.1.3 Stability of the System

2.1.3.1 Demand

Under normal economic circumstances, during periods of continuity in the consumer life cycles (see Maybury, 2020 [Paper 00, §3.1.7]) and stability in their financial circumstances (see Maybury, 2020 [Paper 01]), regular financial behaviour is based on habit. Habit alleviates mental effort and saves time in performing regular consumer activities (Maybury, 2022 [Book 01, Preprint B1-1-2]). Given that decision-making requires calorific energy, for the most trivial decisions, it would be too mentally tiring for consumers to seek to maximise the benefit to cost ratio of every regular purchase. Moreover, most people do not have the mathematical aptitude to do this activity, even for major one-off purchases. Habit is often described as loyalty and a benefit is to reduce purchasing risk, i.e., to avoid disappointment.

All goods, services, and assets, with a few legal constraints, can be sold to any legal entity within or between economies (Maybury, 2020 [Paper 02]). This means that businesses are consumers, as well as households. To understand the demand on supplies, which is reveal by supplies selling out or remaining unsold, then we must account for the demand from both households and businesses. The stability of business demand is like that of households. Businesses tend to have a list of approved suppliers, and often within this list, there is a hierarchy of supplier preference. For an established business with a mature product range, there is little incentive to risk changing reliable suppliers. They will regularly review the supplier list, but rarely change the preferred suppliers. Reviewing all potential suppliers for each order of input supplies is impractical in the amount of effort to review the options and ensure that the appropriate quality of supplied input goods is delivered. So, to ensure cost efficiency of the procurement processes and minimise purchasing risk, these lists which have supplier preference hierarchy, means there are generally go-to providers of a particular set of supplies.

If the demand for a supply is not satiated, then a proportion of the unfulfilled demand may carry over to the next simulated time interval. If a business is seeking input goods to fulfil an order book, then it may carry over some of its demand into the next trading period. Similarly, if someone is seeking a particular shoe or garment that is out-of-stock, they may place an order and wait until new supplies are available instead of purchasing an alternative product. If a household is replenishing their store cupboard goods, they may wait for their preferred brands to become available. In this way, some unfulfilled demand could be carried over.

2.1.3.2 Catchment area constrains demand

A consumer can purchase a supply, for most goods and services, from several suppliers. Any supplier able to fulfil the demand is within the demand catchment area. For food staples (or essentials) the demand catchment area will usually depend on the distance to the retailer by foot, or by public transport, or by car from the residence of the consumer, as well as any suppliers able to deliver to that location. Therefore, the demand catchment for food necessities will be a combination of distance from the consumer location, with distance weighted by convenience against cost, including any delivery service options. The nature of the catchment will depend on the good or service, such as a loaf of bread has a different catchment area to the sale of a car. This can be captured in collecting historical data of consumer behaviour, to determine consumer preferences under different economic circumstances.

2.1.3.3 Effects of price

A change in the price of supplies affects consumer behaviour for both households and businesses and this is a topic that requires research. Unfortunately, under current economic theory it is mistakenly believed that consumer behaviour, which is limited only to households, affects price, and that all consumer behaviour is a choice (as claimed by utility theory). So, the effect of a price rise of a regular purchase above a threshold inducing a change in consumer behaviour, has not been quantified in public literature.

Under the framework proposed here, it is hypothesised that the effect of price on behaviour will be consistent amongst subgroups of consumers within an economics subclass. This is described for households in Maybury, 2020 [Paper 01] and will be described for businesses in Paper 05. Under this hypothesis, by collecting population data, under a framework that can be verified, it will be possible to collate historical data to determine trends in the change of financial behaviours between the different subgroups of consumers.

The data of a change in consumer financial behaviour due to price could be collated in a manner like typical market research by questioning random samples of the public, or by people participating in monitored financial activity (in return for rewards that do not affect the data). Then within a simulation the change in behaviour driven by price can be determined by the population data. However, this data may already be known, to some extent, by supermarkets from their loyalty card data collection schemes, which will be used to position and sell their own branded products in an effective, profitable approach.

Once we have collated population data, the change in consumer behaviour can be represented by using surrogate models. The term *surrogate model* is a general name for the group of analytical methods that have arbitrary pattern capturing properties, from regression fit statistical models (such as general linear models) to machine learning algorithms (such as artificial neural networks). All surrogate models have parameter values populated by training or fitting the model to an existing dataset. Once fitted the model can 'predict' the response of the system that produced the training dataset for a given set of input variable values. Ideally the input variable values must be within the data range of the historical dataset, thus using the surrogate model for interpolation and not extrapolation. In representing the change in consumer behaviour due to price, it is proposed that research collate the data necessary to build surrogate models.

2.1.3.4 Production

A retailer selling out of a good represents a potential loss of sales. Therefore, there should always be a residual supply on the shelves available for purchase. This means that producers, supplying the retailers, aim to match their newly produced supplies to their expected, historical customer demand for those supplies. The stability of demand ensures a consistent baseline for production rates and subsequent sales.

Once the supply output of a business is scaled to meet an expected historical demand, such as having the necessary equipment and staffing levels, there will be scope to decrease or increase production within a small range, without incurring risk or significant financial costs. This is known as an elastic expansion or contraction of production. An elastic expansion of production increases the volume of output, in the short-term, by paying overtime to existing employees or hiring subcontractors. An elastic contraction of production would end any overtime and dismiss contractors. An elastic change in production provides stability to the availability of supplies within a timely manner.

When expansion or contraction of production is necessary above the elastic potential, then the change in production is inelastic. In the case of the upscaling of production, there will be associated financial risks for asset, infrastructure, and work force change implementation, which will also delay the additional demand fulfilment. In contrast, downscaling of production may impact business viability. When stable the producer will have a small residual stock at the end of each trading period, and they will replenish during each period.

A proportion of the unsold supplies may carry over to the next simulated time interval, such as the proportion that is within a sell-by date for perishable items.

The theory must resolve the supply levels and demand levels, in a form that is representative of the business decision-making metrics, which will be associated with profit and loss reporting. Thus, we must determine the rate at which unsold stock accumulates or the rate at which stock sells out.

2.1.4 BUSINESSES HAVE INCOMPLETE INFORMATION

Consumer choice will be consistent with that of previous periods. However, when the market experiences disruption, from a commercial, a political or an environmental event, then there will be a consumer selection process for alternative goods and services (Maybury, 2020 [Paper 01]). Generally, for an established consumer, their sequence of preference may include retail store, product brand,

and product alternative. This information can be obtained from population data as a preference probability and will be different for different economic classes of consumers.

The options available to the consumer, include:

- Change the brand within the same store.
- Change the store but keep the brand at a better price.
- Change the store and change brand such as choosing the own brand of a new store.
- Select a product alternative, such as rice instead of pasta.
- Go without.

In decision-making we must consider how the consumer knows about the opportunity to save money or the need to change store, brand, or product. Information within the system about the sales price of supplies and the suitable alternative products is incomplete, and must be received through advertising, word of mouth, or by seeking alternatives due to a lack of availability of the regular product.

Supermarkets (physical and online stores) enable price comparisons between the products sold by that retailer. However, information about the price of purchasing similar products from different retailers is generally only received through advertising. Some consumers may choose between brands on price alone, often within a preferred supermarket. Many consumers have a price range tolerance built into their choice preferences, known as brand loyalty. A competitor product would need to fall below a threshold price to cause consumers to switch from the regularly purchased brand. Similarly, if their regular brand increases its price above a threshold price, then consumers may also select an alternative brand or product.

Information about the system (demand, supply, and prices) is incomplete. Indeed, markets can either have distributed information, such as bartering in a street market, or have centrally managed information, like an auction. To understand the effect of information about supply and demand on the price, we must determine the information available to suppliers and consumers, and how they receive this information. This is broadly through either a distributed or a centralised market mechanism.

Distributed Markets: Numerous independent consumers and several independent suppliers. Information about supply and demand is revealed either through the established supply chain, or by a drop in sales during a given reporting period. Therefore, in the simulation framework the supply and demand relationship must be resolved after an appropriate time lag following the production and sales (or failure in sales) of supplies. This will then elicit a decision-making response by the producers and retailers.

Centralised Markets: An auction type system. The price is based on consumer competition, with the price being an arbitrary increment larger than the price offered by the highest unsuccessful bidder. In an auction it doesn’t matter how many people attend, the price will only be determined by two people, namely the highest bidder and the second highest bidder. This can be established in the simulation from calculating the financial circumstances of the household and business consumers and identify the relevant subgroups whose buying power will set the price. However, the current central markets are controlled by intermediaries, with artificial price setting processes, that can be modelled given the process definitions.

2.2 Representing the System

2.2.1 Price constraints from the business perspective

The price setting constraints and the resultant retailer decision-making, from the perspective of a business, are presented as a mathematical exposition in Section 3. The structure of the methodology is summarised as follows:

- The retail price is the cost price plus a profit margin (Section 3.1).
- The cost price of production is unique to a company (Section 3.2).
- The profit margin has business needs in its distribution (Section 3.3).
- Customers pay the gross price and not the retail price (Section 3.4).
- The gross price must not be significantly undercut by competitors (Section 3.5).
- Companies must make a minimum profit to be viable (Section 3.6).
- Business decision-making is refined by sales history (Section 3.7).

The framework in Section 3 incorporates feedback mechanisms, such as sales performance, to provide a methodology for evaluating and optimising pricing decisions under various scenarios. By formalising the decision-making process, the model has application for understanding and enhancing retail pricing strategies within complex economic environments. Moreover, it offers a structured methodology for businesses to navigate the complex landscape of retail pricing.

2.2.2 Resolving the price of supplies in a computational framework

In an economy, the determination of supply price transcends simple market equilibrium theories, necessitating a dynamic, multidimensional approach. Section 4 introduces a detailed mathematical framework designed to simulate price determination and market behaviour, extending the methodology in Section 3 into a large-scale computational framework. Specifically, it provides the functions for resolving the price of supplies within the Economy Dynamics framework.

Resolving the price of supplies has a significant number of datasets to define, which are to be processed by numerous equations. Therefore, before expressing the equations of retailer decision-making, the data structures (i.e., the system) are defined (Section 4.1). Then, the approach to resolving the price of supplies follows the logic below:

Step 1. Calculate the monetary and non-monetary cost price of supplies (Section 4.2).
Step 2. Estimate the unit profit margin and set the initial unit retail price (Section 4.3).
Step 3. Compare to competitor prices from the previous time interval (Section 4.4).
Step 4. Set minimum profit margin to meets the business needs (Section 4.5).
Step 5. Check if the pricing is stable (Section 4.6).

If stable, no further calculations are necessary, otherwise the following steps are required to resolve sales feedback:

Step 6. Evaluate the effect of retail price on returning customer behaviour (Section 4.7).
Step 7. Resolve the rate of stock selling out or remaining unsold (Section 4.8).
Step 8. Identify if production can be reduced/increased at competitive costs (Section 4.9).

The methodology in Section 4 uses surrogate models to represent complex relationships from observed data, incorporating probabilistic elements. The aim is to create a robust and realistic simulation of economic systems to predict outcomes under varying conditions. This approach facilitates the analysis of current supply dynamics and enables predictive simulations under shifting economic conditions.

The model also accounts for both monetary and non-monetary costs (like greenhouse gas emissions). Its forecasts will be repeatable, achieving the scientific requirement of falsification (see Section 5.3) through its validation against observed features of an economy, such as the change in rate of inflation. This is an important feature leading to determine the effect of economic policies on features of an economy, such as the change to the cost-of-living, future tax revenues and environmental sustainability.

3 PRICE CONSTRAINTS

This section introduces a model for determining the retail price of goods produced by a business, representing the production costs and the profit margin decision-making. It presents the framework from a business perspective. It defines the production cost by accounting for factors such as input supply costs, labour rates, taxes, and asset leasing fees. Then model addresses critical constraints, including the concept of minimum viable profits, the influence of competitor pricing and the necessity of balancing profitability with market competitiveness. The methodology starts with the calculation of an initial retail price.

3.1 *RETAIL PRICE*

To set the retail price p_r (excluding sales tax), the business sales manager applies a mark-up factor to the cost of production, so that,

$$p_r = p_c(1 + r_S)\,, \qquad 1$$

where:

p_c	is the unit cost price of production, Equation 4.
r_S	is the unit *sales* mark-up factor.

There are several different calculations of profit margin. The one discussed here is based on the *net* profit margin[2]. In addition, by assuming all units produced are sold, the relationship between the unit retail price and cost price is,

$$p_r(1 - \tilde{r}_\beta) = p_c\,, \qquad 2$$

where:

$\tilde{r}_\beta$	is the estimated unit *business* profit margin based on the sales mark-up factor.

The relationship between the sales mark-up factor and the net business profit margin, assuming all units produced are sold, is,

[2] Typically, the markup would factor the gross cost price of production, which excludes the cost of taxes and debt repayment. In contrast, the net cost price of production is the value used to determine the bottom-line of company accounts: the money that can be distributed to shareholders. This paper breaks from convention with the mark-up factor because the net cost of production is more critical in understanding the income of worker-owners. For example, the calculation of net profit is required and not gross profit, and so a revised form of the mark-up factor to estimate the net profit margin is needed.

$$\tilde{r}_\beta = 1 - \frac{1}{1+r_S}. \quad 3$$

It is a common mistake to think that a mark-up of, say, 25%, will produce a profit margin of 25%. This is not correct, as a 25% markup rate produces a profit margin of 20%, as shown by Equation 3. The definition of the *business* profit margin is discussed further in Section 3.3.

3.2 COST PRICE

The initial retail price is dependent on factoring the unit cost price of production p_c which is calculated as,

$$p_c = \frac{q_{p_c}}{s_{ns}}, \quad 4$$

where:

q_{p_c}	is the total cost, or net cost of production, Equation 5.
s_{ns}	is the number of units of the new supply produced.

In turn, the total cost or net cost of production q_{p_c} is,

$$q_{p_c} = q_{\Sigma ds_\beta} + q_{y_{\alpha_L}} + q_{\Sigma ds_A} + q_{T_{\beta_{AL}}} + q_{\Sigma ds_D} + q_{T_{\beta_{CT}}}, \quad 5$$

where:

$q_{\Sigma ds_\beta}$	is the cost of input supplies including duties and taxes, Equation 6.
$q_{y_{\alpha_L}}$	is the payment of wages for labour including employment taxes, Equation 7.
$q_{\Sigma ds_A}$	is the fee for land usage and other leasing services, Equation 8.
$q_{T_{\beta_{AL}}}$	is the cost of local business taxes, based on land usage, Equation 9.
$q_{\Sigma ds_D}$	is the cost of debt repayment, Equation 10.
$q_{T_{\beta_{CT}}}$	is the cost of corporate tax, Equation 14.

Definition of the variables that comprise the operational cost are detailed respectively below.

The cost of input supplies $q_{\Sigma ds_\beta}$ (demand by a business, supplied by another business) is,

$$q_{\Sigma ds_\beta} = \sum_{j_b=1}^{n_b} d_{\beta(j_b)} \left(p_{r(j_b)} \left(1 + r_{T_V(j_b)} \right) \right), \quad 6$$

where:

j_b	is the input supply category.
n_b	is the number of input supply categories.
d_β	is the number of units demanded (purchased).
p_r	is the retail price for the input supplies.
r_{T_V}	is the duty and value added tax.

The payment of wages $q_{y_{\alpha_L}}$ is,

$$q_{y_{\alpha_L}} = \sum_{j_o=1}^{n_o} d_{L(j_o)} y_{\alpha_L(j_o)} \left(1 + r_{\beta_{T_\gamma}(j_o)} \right), \quad 7$$

where:

j_{o}	is the occupation category including seniority and experience.
n_{o}	is the number of occupation categories.
d_L	is the effort, in terms of the number of wages or salaries consumed in production.
y_{α_L}	is the wage or salary.
$r_{\beta_{T_\gamma}}$	is the rate of employment tax including both employee and employer taxes.

The cost of rent and asset leasing $q_{\Sigma ds_A}$ is,

$$q_{\Sigma ds_A} = \sum_{j_{b_A}=1}^{n_{b_A}} d_{A(j_{b_A})} \left(p_{r(j_{b_A})} \left(1 + r_{T_V(j_{b_A})}\right)\right), \quad 8$$

where:

j_{b_A}	is the category of asset leased (and not owned).
n_{b_A}	is the number of asset categories.
d_A	is the property floor space and/or the number of asset units leased.
p_r	is the leasing unit retail price.
r_{T_V}	is the sales tax and duty rates.

The cost of local business tax $q_{T_{\beta_{AL}}}$ based on land usage by using method of the UK is,

$$q_{T_{\beta_{AL}}} = d_{AL} p_{r_{AL}} r_{\beta_{T_{AL}}}, \quad 9$$

where:

$d_{AL} p_{r_{AL}}$	is the retail rental value of the business premise.
$r_{\beta_{T_{AL}}}$	is the business tax rate.

The cost of debt repayment $q_{\Sigma ds_D}$ is,

$$q_{\Sigma ds_D} = \sum_{j_{b_D}=1}^{n_{b_D}} f_{D(j_{b_D})}(-z_D, r_D, t_D), \quad 10$$

where:

j_{b_D}	is the category of debt.
n_{b_D}	is the number of debt categories.
$f_D(\ )$	is a function of $-z_D, r_D, t_D$, such as Equation 11.
z_D	is the money owed.
r_D	is the annual interest rate on the debt.
t_D	is the duration of the repayment schedule.

The reason for representing a function as $f_{D(j_{b_D})}$ is that all loans to businesses have well-defined methods for calculating the repayment schedule. The functions and types of debt are briefly discussed in Maybury 2020 [Paper 01, §6.2.1]. An example of a typical debt function, familiar to households with a repayment mortgage is,

$$f_{D(j_{b_D}=1)}(-z_D, r_D, t_D) = -z_D \frac{\Delta r_D(1+\Delta r_D)^{n_P}}{(1+\Delta r_D)^{n_P}-1}, \quad 11$$

where:

j_{b_D}	is the category of debt that determines the function.
Δr_D	is the interest rate for each time interval of the debt repayment, Equation 12.
n_P	is the number of repayments, Equation 13.

and where the interest rate Δr_D for the model time step of the debt repayment is,

$$\Delta r_D = r_D \frac{\Delta t}{365}, \quad 12$$

where:

Δt	is the incremental timestep of the simulation (days),

and where the number of repayments n_P is,

$$n_P = \frac{t_D}{\Delta t}, \quad 13$$

where:

t_D	is the duration of the loan.

The cost of corporate tax $q_{T_{\beta_{CT}}}$ is,

$$q_{T_{\beta_{CT}}} = q_{r_\beta} r_{\beta_{T_{CT}}}, \quad 14$$

where:

q_{r_β}	is the total taxable business profit, Equation 26.
$r_{\beta_{T_{CT}}}$	is the rate of corporate tax.

3.3 PROFIT MARGIN

To establish the relationship between the estimated profit margin and net profit margin per unit supply, we start by defining the total net business profit margin r_β as,

$$r_\beta = \frac{q_{p_r} - q_{p_c}}{q_{p_r}}, \quad 15$$

where:

q_{p_r}	is the total sales revenue from the sale of goods and services, Equation16.
q_{p_c}	is the total production cost, obtained by rearranging Equation 4.

The total sales revenue q_{p_r} is,

$$q_{p_r} = p_r s_d, \quad 16$$

where:

p_r	is the retail price per unit of supply, Equation 1.
s_d	is the number of units of supply sold (demanded).

The profit margin per unit supply r_β is then established by combining Equation 15 with Equations 4 and 16, such that,

$$r_\beta = \frac{p_r s_d - p_c s_{ns}}{p_r s_d} . \tag{17}$$

The business sales manager decision of setting the retail price assumes that $s_d = s_{ns}$, the number of units sold are all the units produced. Company accounting aligns the number of supplies produced with them all being sold, and it achieves this by estimating the value of the unsold supplies, to be sold in the future, and the spoilt and faulty supplies, to be included in the cost of production. By assuming that $s_d = s_{ns}$, then the estimated business profit margin $\tilde{r}_\beta$ is,

$$\tilde{r}_\beta = \frac{p_r - p_c}{p_r} . \tag{18}$$

For modelling purposes, the estimated profit margin of a business can be initially calculated from the historical data of the business, such that,

$$\bar{r}_{\beta(k_t)} = \frac{1}{n} \sum_{i=1}^{n} r_{\beta(k_t - i)} , \tag{19}$$

where:

k_t	is the timestep needing an estimated value of profit margin.
n	is the number of historical values of profit margin used in the calculation.
i	is the number of years into the past.
r_β	is the historic value of profit margin for the business.

Then the modelled retail price can be determined by substituting the averaged historical profit margin in Equation 19 with the estimated business profit margin in Equation 2, assuming that $\bar{r}_\beta \cong \tilde{r}_\beta$, and rearrange for until retail price p_r, so that,

$$p_r = \frac{p_c}{1 - \bar{r}_\beta} , \tag{20}$$

where:

p_c	is the unit cost price of production, a known actual unit cost, Equation 4.
$\bar{r}_\beta$	is the estimated unit *business* profit margin from historical data, Equation 19.

The retail price that has been calculated must meet additional criteria, before being offered to customers. These criteria are based on the price of competitors, the minimum viable profits and recent sales feedback, which are discussed in the following sections.

3.4 *Gross Price*

The retail price is not the price the consumer pays. This is the gross price per unit supply p_x, which includes duties and value added taxes,

$$p_{x(j_b)} = p_{r(j_b)} \left(1 + r_{T_V(j_b)}\right) , \tag{21}$$

where:

j_b	is the input supply category.
p_r	is the retail price per unit of supply, Equation 1.
r_{T_V}	is the duty and value added tax.

The cost of tax p_T is,

$$p_{T(j_b)} = p_{r(j_b)} r_{T_V(j_b)} \ , \qquad 22$$

where:

j_b	is the input supply category.
p_r	is the retail price per unit of supply, Equation 1.
r_{T_V}	is the duty and value added tax.

3.5 COMPETITOR PRICES

The initial retail price defined by the sales manager, by factoring the cost price, will be compared to the prices in the market, as part of a business judgement on the positioning of their output supplies within a relative price range.

When the business competes directly on price, and price alone, then the supply gross price p_x should,

$$p_{x(k)} \leq min\left(p_{x(\boldsymbol{j})}\right) , \qquad 23$$

where:

k	is the company setting their unit retail price.
$\boldsymbol{j}$	is the list of direct competitors.

If the inequality in Equation 23 is false, then ideally the retail price must be adjusted so that the inequality becomes true, i.e., $p_{x(k)} = \min\left(p_{x(\boldsymbol{j})}\right)$. This must include any differentiation in the treatment of taxes.

When the business has brand loyalty amongst its customer, then the gross price must be,

$$p_{x(k)} \leq min\left(p_{x(\boldsymbol{j})} + \partial p_{x(\boldsymbol{j})}\right) , \qquad 24$$

where:

∂	is the price tolerance of existing customers.

If the inequality in Equation 24 is false, then the retail price must be adjusted so that the inequality becomes true, i.e., $p_{x(k)} = \min\left(p_{x(\boldsymbol{j})} + \partial p_{x(\boldsymbol{j})}\right)$.

For businesses that do not have a dependency on their output supply retail price imposed by the retail price of its competitors, retail price is independent $\perp$ (orthogonal) of the other gross prices in the market, such that,

$$p_{x(k)} \perp p_{x(\boldsymbol{j})} \ . \qquad 25$$

These equations explain why there is a tendency for mature markets to consolidate to only a few major players. Namely, businesses for which Equation 25 is applicable determine the viability of the businesses for which Equations 23 or 24 are applicable.

3.6 MINIMUM VIABLE PROFITS

For any businesses that are constrained on their retail prices, by either Equation 23 or Equation 24, they must verify that it is possible to reduce the retail price, whilst maintaining a viable profit margin. The profit margin must meet business needs, such as for most businesses the dividends paid to worker-owners must cover their cost-of-living. If this is the case, then businesses will take the risk of making the inequality in Equation 23 or Equation 24 false. i.e., it will set a price above its competitors.

Some business decisions are based on the value of profits before tax. To reduce the company's corporate tax bill, business directors make judicious financial transactions that can be offset against tax, such as buying new equipment. Similarly, some business activities will be constrained if the company's profitability is low, such as undertaking research and development. This means that the profit margin will be calculated once, to determine the availability to perform further spending, and then it will be recalculated and submitted to the tax office, accounting for the additional spending (see Equation 15).

3.6.1 PROFIT MARGIN AND BUSINESS REVENUE

To understand the constraints on the allocation of the profit margin, consider that the total profits achieved by a business q_{r_β} is,

$$q_{r_\beta} = y_\beta r_\beta \,, \tag{26}$$

where:

y_β	is the business revenue excluding the value of assets traded, Equation 27.
r_β	is the business profit margin, Equation 15.

and where the business revenue y_β is,

$$y_\beta = q_{\Sigma sd} + y_{\beta_B} + y_{\beta_A} \,, \tag{27}$$

where:

$q_{\Sigma sd}$	is the retail value of supplies sold excluding taxes (supplies demanded), Equation 28. It includes asset trading / leasing, as $q_{\Sigma sd_A} \subset q_{\Sigma sd}$, and the lending of money, as $q_{\Sigma sd_D} \subset q_{\Sigma sd}$. The trading and leasing of assets and lending money for profit will be discussed in Paper 06 (in progress).
y_{β_B}	is the receipt of all benefits, such as government subsidy of public sector businesses and government grants to private businesses.
y_{β_A}	is the receipt of dividends or royalties from asset ownership, including intellectual property.

The values of $q_{\Sigma sd}$, y_{β_B}, and y_{β_A} are the sum of the respective units of supply, benefit payments, and dividends and royalties. We will neglect the business income from government benefits and dividends, except to note that these indirectly affect the cost price of production by offsetting business operational costs[3].

The retail value of all supplies sold $q_{\Sigma sd}$ is,

$$q_{\Sigma sd} = \sum_{j_b=1}^{n_b} s_{d(j_b)} p_{r(j_b)} \,, \quad 28$$

where:

j_b	is the supply model produced, i.e., differentiating between products produced.
n_b	is the number of different models produced.
p_r	is the retail unit price excluding tax, see Equations 23, 24, and 25.
s_d	is the number of units of supply sold (supplies demanded).

For a business that only produces one model, $n_b = 1$, then $q_{\Sigma sd}$ from Equation 28 is equivalent to q_{p_r} from Equation 16. The effect of businesses retailing a large range of products and services, is that they have scope to manipulate the retail price of some goods and services, to change consumer behaviour. This explains the lost leader model of supermarkets, whereby retailers reduce the price of a staple good to undercut competitor retailers, to increase consumer footfall, whilst making the required overall business profit from the sales of other goods. Thus, the estimated profit margin $\tilde{r}_\beta$ in Equation 18 may vary between products retailed by a business.

3.6.2 Allocation of profits

As discussed in Section 2.1.1.1, in relation to owner-workers, businesses have a need for profits, and they must be allocated according to that need. The allocation of business profits q_{r_β} is,

$$q_{r_\beta} = y_{\alpha_B} + y_{\beta_B} + m_\lambda + q_{\Sigma d}, \quad 29$$

where:

y_{α_B}	is the payment of dividends to the household owners of the business.
y_{β_B}	is the payment of dividends to the business owners of the business.
m_λ	is the money retained by the business for cash-flow management.
$q_{\Sigma d}$	is the money reinvestment to maintain and grow the business (one-off purchases).

These variables have different effects on the duration a business can manage an inadequate profit margin. The business may try to maintain a sufficient profit margin by setting their retail price higher than their competitors (or above their competitors plus a brand loyalty tolerance). The feedback from the sales results will determine further business decisions.

The consequence of reduced profits for owner-workers (y_{α_B}) is dependent on their flexibility to meet their cost-of-living when the dividends to households fall. If they have savings, or can borrow money at personal risk, they can offset the loss in the short-term. This situation is not sustainable long-term.

[3] There may be circumstances when government grants and subsides substantially affect the profits of the private sector. If this is important for the analysis, then this must be considered. An example of the non-negligible effects of government intervention on private sector profitability is shown by the current electric vehicle sector. See, for example, Isidore (2024).

The consequence of reduced profits for owner-investors for both household (y_{α_B}) and business investors (y_{β_B}), is a risk to the board of directors, that they will be replaced.

The consequence of reduced profits is an increased cashflow vulnerability to delays in receipt of payments from customers and the need to meet outgoing costs when the retained money m_λ cannot be replenished after each sales period. This exposes the business to a greater impact when there are delays in receipt of customer payments and foreclosing in the demands of payments to suppliers or the need to replace worn or faulty equipment.

The consequence of reduced profits may be a reduced ability to replace worn equipment or invest in keeping up with market development ($q_{\Sigma d}$), such as the implementation of automation to reduce production costs. This can be address by securing a business loan, but this will contribute to an increase in the cost of production (net cost).

3.7 SALES FEEDBACK

Ultimately the decision on the profit margin will be determined by sales feedback: the inequality of the supplies produced to those demanded (sold). The set of business decisions depends on the magnitude of the inequality and the timeframe of response of selling out or stock remaining unsold. Within each decision-making period there are five scenarios for the level of supplies to the level of demand:

1) When supply exactly matches demand, there is a small residual stock at the end of each trading period,

$$S_{ns} = D_{nd} \, , \quad (30)$$

where:

S_{ns}	is the volume of supplies produced in a reporting period,
D_{nd}	is the volume of demand in a reporting period seeking to purchase the supply.

2) When supply is smaller than demand, stock sells out before it is replenished,

$$S_{ns} < D_{nd} \, . \quad (31)$$

3) When supply is much smaller than demand, stock sells out quickly leaving empty shelves,

$$S_{ns} \ll D_{nd} \, . \quad (32)$$

4) When supply is larger than demand, stock remains unsold and gradually accumulates,

$$S_{ns} > D_{nd} \, . \quad (33)$$

5) When supply is much greater than demand, stock is largely unsold and rapidly accumulates,

$$S_{ns} \gg D_{nd} \, . \quad (34)$$

The ideal situation is that the supplies produced meet the demand (Equation 30), whereby the same level of residual stock remains before the next business cycle replenishes stock. If stock is unsold (inequalities of Equations 31 and 32) the reducing the profit margin must be considered. If stock sells out (inequalities in Equations 33 and 34), and the profit margin had previously been constrained, then there is scope to increase the profit margin.

4 RESOLVING THE PRICE OF SUPPLIES

This section extends the methodology in Section 3 into a large-scale computational framework. It uses sophisticated indexing systems to track goods and services, businesses and households, and economic regions over time, enabling detailed analysis of retailer and consumer interactions. Central to the model are the supply and demand objects, complex data structures that represent the flow of goods and services through producers, retailers, and consumers, accounting for factors like production costs (including wages, land usage, and taxes), retail markup, and consumer price sensitivity. By utilising discretised equations and index notation, the framework enables systematic analysis of the relationships between businesses, households, and economic regions, providing an adaptable tool for simulation and analysis. The methodology starts with the definition of these data structures.

4.1 THE DATA STRUCTURE

The equations in this section are written in a discretised form that includes temporal feedback between economic regions and business categories in the price of input supplies, which is in turn affected by the cost of wages, land usage fees, and business operational taxes. Therefore, the system must be defined, which is achieved through an indexing methodology.

4.1.1 DEFINING THE SYSTEM

4.1.1.1 Indices

In computer programming, index notation is used to specify the position of a variable value stored within a list or array. In Economy Dynamics indices are used to identify what (goods, services, and assets) and when (a duration between given dates) and where (economic regions) and who (businesses and households). These are discussed in turn below (Sections 4.1.1.1.1 to 4.1.1.1.4).

The index $\boldsymbol{i}_{\dots}$ is used to list all items within an economy. For example, $\boldsymbol{i}_b$ represents all goods, services and assets sold within the simulation. The index $\boldsymbol{j}_{\dots}$ is a list of items determined by another know input, so that $\boldsymbol{j}_{\dots}$ is a subset of $\boldsymbol{i}_{\dots}$, such that $\boldsymbol{j}_{\dots} \subset \boldsymbol{i}_{\dots}$. Therefore, $\boldsymbol{j}_b$ can be used to represent the list of input goods, services and assets required by a category of business. Moreover, depending on the context, $\boldsymbol{j}_b$ can be used to store a different list, such as the list of inputs goods, services and assets for other categories of business. The index $k_{\dots}$ is a specific index value determined by another know variable, such that that $k_{\dots} \subset \boldsymbol{i}_{\dots}$. For example, k_b could represent a single category of supply (a type of good or service or asset). Note that an index that represents a single value is formatted in italics (i.e., $k_{\dots}$), whereas an index list (index vector) is formatted in bold and italics (i.e., $\boldsymbol{j}_{\dots}$ or $\boldsymbol{i}_{\dots}$).

Conventionally, in computer science and engineering, the number of items within an index vector, known as the length of the vector, is stored by using the integer $n_{\dots}$ or $N_{\dots}$. The distinction between lowercase and uppercase variables in this paper (excluding $\boldsymbol{i}$, $\boldsymbol{j}$ and k) relates to the whether the variable uses a non-probabilistic or a probabilistic approach to its definition. For example, the lowercase $n_{\dots}$ represents the length of the vector list of items (such as the number of individual households listed). In contrast, the uppercase $N_{\dots}$ represents the length of the vector list of a probabilistic grouping, such as the number of household subgroups (households grouped by similarity of financial behaviour) within an economic region. The use of lowercase and uppercase variable is discussed further in Section 4.1.3.

4.1.1.1.1 Supplies (what)

The list of all goods (including commodities), services, depreciable assets, financial products, land, property, equipment, private business stake-holding and intellectual property traded within the economy is represented by the index $\boldsymbol{i}_b$. The definition of supplies follows a similar ontology to the consumer basket of goods and services defined by the Office for National Statistics (ONS.Gov.UK, 2019 [1] and [2]). However, the basket of supplies is for the whole economy, including undeclared, non-tax paying, aspects. A more detailed description is provided in Maybury, 2020 (Paper 01, §5.2) and the comprehensive definition will be provided in Paper 09 (in progress).

The index $\boldsymbol{i}_b$ comprises a ragged array of three indices that allow the aggregation and disaggregation of categories, depending on the objective of the simulation, such that,

$$\boldsymbol{i}_b \equiv [\boldsymbol{i}_{1b}, \boldsymbol{i}_{2b}, \boldsymbol{i}_{3b}], \quad 35$$

where:

- $\boldsymbol{i}_{1b}$ is the supply superclass, e.g. food | clothing | furniture | education |... ;
- $\boldsymbol{i}_{2b}$ is the supply class, e.g. from food superclass cereals | meat | fish |... ;
- $\boldsymbol{i}_{3b}$ is the supply subclass, e.g. from cereals class bread | rice | pasta | ... .

The ragged array means that the length of the list of items in $\boldsymbol{i}_{2b}$ is dependent on the index value of $\boldsymbol{i}_{1b}$ and, similarly the length $\boldsymbol{i}_{3b}$ is dependent of the index value of $\boldsymbol{i}_{2b}$. So, the number of supply subclasses or food, clothing, furniture, education can be different, and the number of subclasses of cereals, meat and fish can be different.

When aggregating supplies and assets every superclass must have at least one class, and every class must have at least one subclass. For example, to aggregate all food into a single superclass, it must contain only one class, which in turn must contain only one subclass. In this case, the model would not distinguish between any food types. This is an extreme example, as it would mean there was no distinction between cereals, fish, meat, fruit or vegetables in the simulation.

As the structure (Equation 35) is a ragged array, we can choose which superclasses to have coarsely represented by aggregating their classes and subclasses. Similarly, any superclass can be disaggregated by increasing the number of respective classes, and/or increasing their respective number of subclasses. However, disaggregation is constrained by the ability to represent the outcome of a probability, such as the proportion of households in a socio-economic subgroup failing to purchase a necessity (supply).

Note: only the supplies index $\boldsymbol{i}_b$ comprises superclasses, classes and subclasses. All other indices discussed below (when, where and who) are a single list. Therefore, aggregation or disaggregation of these indices simply changes the length of the list.

4.1.1.1.2 Time intervals (when)

Time is a continuous variable t. Within a simulation, a specific duration of time Δt, which marches forwards through progressive increments is denoted by the index $\boldsymbol{i}_t$. For example, if the analyst selects weekly intervals and the simulation starts at 1st week of 2020, then this week is $i_t = 1$ and the 2nd week of 2020 is $i_t = 2$ and so on, to say, the last week of 2020 being $i_t = 52$.

4.1.1.1.3 Economic regions (where)

The subdivision of an economy into distinct economic regions is necessary as production costs of goods and services is affected regionally by the cost of land, local taxes, labour rates and the distribution of debt and asset ownership amongst the population and businesses of the different regions. These regions are termed *economy elements* within the Economy Dynamics framework, but for readability will also be referred to as economic regions in this paper. These regions are dividend based on taxation and living costs for households $\boldsymbol{i}_\alpha$, and taxation and running costs for private $\boldsymbol{i}_\beta$, public $\boldsymbol{i}_\tau$ and charitable $\boldsymbol{i}_\phi$ businesses.

4.1.1.1.4 Businesses and households (who)

4.1.1.1.4.1 Businesses

Businesses are non-human legal entities that produce goods or services in exchange for money. Businesses include all supply producing organisations in both the private, public and charity sectors that can be either for-profit or not-for-profit organisations.

The business population of an economy element is divided in subgroups $\boldsymbol{i}_{B+}$ that have a similar business category based on output supplies and are further subdivided by the enterprise class, the stage of business operations based on the business age, brand differentiation subgroups based on supply quality and ethics of business operations and multi-element indexing for large national and multi-national organisations. Business subgroups will be defined in detail in Paper 05 (in progress).

As businesses are both a consumer and a supplier, then for clarity when a business is involved in the demand side its index has superscript *D* becoming $\boldsymbol{j}_{B+}^{D}$, whereas a supply side business has a superscript *S* becoming $\boldsymbol{j}_{B+}^{S}$. Businesses are subdivided into private sector $\boldsymbol{j}_{B+_\beta}$, public sector $\boldsymbol{j}_{B+_\tau}$, and charity sector $\boldsymbol{j}_{B+_\phi}$. Collectively, all business types are represented by $\boldsymbol{j}_{B+}$.

4.1.1.1.4.2 Households

Households are the financial unit of interdependent human legal entities that provide the economy with the resource of labour. Households cannot sell goods they have produced, nor can they sell services they provide; this income is turnover under the self-employed business category. Households can sell assets, including land, property and durable supplies that have depreciable residual value. The main income for most households is wages (salaries) for their labour.

The household population of an economy element is divided in subgroups $\boldsymbol{i}_{e+}$ that have similar standards of living and are further subdivided by likely financial characteristics and behaviour, based on composition of the household. These include, the job skill category of the main income provider, the stage of family and working life based on the age of the main income provider, their minority grouping and their parental living standard. Household subgroups are defined in detail in Maybury, (2020, [Paper 01]).

Households provide labour represented by $\boldsymbol{i}_{e+}^{S}$. Similarly, households demand goods and services represented by $\boldsymbol{i}_{e+}^{D}$. However, they don't demand labour[4], so there is no risk of ambiguity in what is represented by $\boldsymbol{i}_{e+}$, as this is clear from the equation. So, households can be represented without the superscript *S* or *D*. In terms of the supply of labour, we divide households into subgroups by using an occupation index $\boldsymbol{i}_o$, which incorporates level of training and experience, based on the age category of the main income of the household.

4.1.1.2 Consumers and suppliers

Consumers are both households and businesses, whereas suppliers of goods and services are only businesses and suppliers of labour are only households. This section introduces the indexing of consumers and suppliers incorporating all applicable entities within an index that comprises the relevant household and business indexing lists. These are therefore composite lists of the household and business indices. Note that aggregation and disaggregation of the subdivisions of households and businesses changes the length of the relative list within the composite list. Unlike the indexing of supplies, the lists of households and business do not use superclasses, classes and subclasses. It is just a single list. The composite indices distinguish between the consumer side and supply side as follows.

The list of consumer groups:

$\boldsymbol{i}_D$	is the list of all consumer groups, including household and businesses, Equation 36.
$\boldsymbol{j}_D$	is the subset list of consumer groups, Equation 37.
k_D	is the specific consumer group, Equation 38.
$\boldsymbol{i}_{DB}$	is the list of business consumers, without households, Equation 39.
$\boldsymbol{j}_{DB}$	is the subset list of business consumers groups, Equation 40.
k_{DB}	is the specific business consumer group, Equation 41.

The list of business groups, as retailers and producers:

$\boldsymbol{i}_S$	is the list of all business groups as suppliers, Equation 42.
$\boldsymbol{j}_S$	is the subset list of business groups as suppliers, Equation 43.
k_S	is the specific business group as suppliers, Equation 44.
$\boldsymbol{i}_S$	is the list of all business groups as producers, Equation 45.
$\boldsymbol{j}_S$	is the subset list of business groups as producers, Equation 46.
k_S	is the specific business group as producers, Equation 47.
$\boldsymbol{i}_{2S}$	is the list of all business groups as retailers, see Equation 42.
$\boldsymbol{j}_{2S}$	is the subset list of business groups as retailers, see Equation 43.
k_{2S}	is the specific business group as retailers, see Equation 44.
$\boldsymbol{i}_{1S}$	is the list of all business groups as retailers, see Equation 45.
$\boldsymbol{j}_{1S}$	is the subset list of business groups as retailers, see Equation 46.
k_{1S}	is the specific business group as retailers, see Equation 47.
$\boldsymbol{i}_o$	is the list of all labour supply (by occupation and experience), Equation 48.
$\boldsymbol{j}_o$	is the subset list of all labour supply (by occupation and experience), Equation 49.

[4] Employing domestic staff sometimes requires the household to become an employer. To handle this situation, if needed, the household is treated as a non-profit business providing a service. Maintaining the separation between business and household behaviour vastly simplifies the computational framework. It avoids unnecessary complexity created when rules are defined by exception, evident in the numerous loopholes of modern tax systems. The approach follows software development best practices, where algorithms must be coded in a formulation that avoids logical exceptions.

k_o is the specific labour supply (by occupation and experience), Equation 50.

These composite indices are defined in more detail below.

4.1.1.2.1 Consumers (demand)

The list of all consumer categories (subgroups) is represented by the index $\boldsymbol{i}_D$. All entities are consumers, which includes all household subgroups $\boldsymbol{i}_{e+}^D$ in their respective economic regions $\boldsymbol{i}_\alpha^D$, and the business subgroups, which are in the private $\boldsymbol{i}_{B+_\beta}^D$, public $\boldsymbol{i}_{B+_\tau}^D$ and charity sectors $\boldsymbol{i}_{B+_\phi}^D$, within their respective economic regions $\boldsymbol{i}_\beta^D$, $\boldsymbol{i}_\tau^D$ and $\boldsymbol{i}_\phi^D$. This is represented as

$$\boldsymbol{i}_D \stackrel{\text{def}}{=} \begin{Bmatrix} \boldsymbol{i}_\alpha^D, \boldsymbol{i}_{e+}^D \\ \boldsymbol{i}_\beta^D, \boldsymbol{i}_{B+_\beta}^D \\ \boldsymbol{i}_\tau^D, \boldsymbol{i}_{B+_\tau}^D \\ \boldsymbol{i}_\phi^D, \boldsymbol{i}_{B+_\phi}^D \end{Bmatrix}, \qquad 36$$

where the curly brackets $\{\quad\}$ is a composite list of other lists.

Following the notation convention discussed in Section 4.1.1, then a known subset of consumers is represented by $\boldsymbol{j}_D$. This is a list of consumer groups, which is a subset of all consumers $\boldsymbol{i}_D$. This list can include any type of consumer entity in their respective economic region. Thus,

$$\boldsymbol{j}_D \stackrel{\text{def}}{=} \begin{Bmatrix} \boldsymbol{j}_\alpha^D, \boldsymbol{j}_{e+}^D \\ \boldsymbol{j}_\beta^D, \boldsymbol{j}_{B+_\beta}^D \\ \boldsymbol{j}_\tau^D, \boldsymbol{j}_{B+_\tau}^D \\ \boldsymbol{j}_\phi^D, \boldsymbol{j}_{B+_\phi}^D \end{Bmatrix}, \ \boldsymbol{j}_D \subset \boldsymbol{i}_D, \ \boldsymbol{j}_{3D} \equiv \boldsymbol{j}_D. \qquad 37$$

Continuing with the notation convention from Section 4.1.1, then a known consumer group is represented by k_D. This is a consumer group belonging to the set of all consumers $\boldsymbol{i}_D$. This group can only represent one consumer type in their respective economic region and such that

$$k_D \stackrel{\text{def}}{=} \begin{cases} k_\alpha^D, k_{e+}^D \\ k_\beta^D, k_{B+_\beta}^D \\ k_\tau^D, k_{B+_\tau}^D \\ k_\phi^D, k_{B+_\phi}^D \end{cases}, \ k_D \subset \boldsymbol{i}_D, \ k_{3D} \equiv k_D, \qquad 38$$

where the open curly brackets $\{$ is a subgroup from either the household or one of the three business lists.

We can also define the business consumers as

$$\boldsymbol{i}_{DB} \stackrel{\text{def}}{=} \begin{Bmatrix} \boldsymbol{i}_\beta^D, \boldsymbol{i}_{B+_\beta}^D \\ \boldsymbol{i}_\tau^D, \boldsymbol{i}_{B+_\tau}^D \\ \boldsymbol{i}_\phi^D, \boldsymbol{i}_{B+_\phi}^D \end{Bmatrix}, \qquad 39$$

a subset of business consumers as

$$\boldsymbol{j}_{DB} \stackrel{\text{def}}{=} \begin{Bmatrix} \boldsymbol{j}_\beta^D, \boldsymbol{j}_{B+_\beta}^D \\ \boldsymbol{j}_\tau^D, \boldsymbol{j}_{B+_\tau}^D \\ \boldsymbol{j}_\phi^D, \boldsymbol{j}_{B+_\phi}^D \end{Bmatrix}, \ \boldsymbol{j}_{DB} \subset \boldsymbol{i}_{DB} \subset \boldsymbol{i}_D, \ \boldsymbol{j}_{3DB} \equiv \boldsymbol{j}_{DB}, \qquad 40$$

and a specific known business consumer category as

$$k_{DB} \stackrel{\text{def}}{=} \begin{cases} k_{\beta}^{D}, k_{B+\beta}^{D} \\ k_{\tau}^{D}, k_{B+\tau}^{D}, \\ k_{\phi}^{D}, k_{B+\phi}^{D} \end{cases} k_{DB} \subset \boldsymbol{i}_{DB} \subset \boldsymbol{i}_{D}, \;\; k_{3DB} \equiv k_{DB}. \qquad 41$$

4.1.1.2.2 Suppliers (retailers and producers)

All businesses, including public and charity sector, are represented by the index $\boldsymbol{i}_S$. All must be retailers and distributors (also listed by $\boldsymbol{i}_{2S}$) as this includes business-to-business retail, i.e., any manufacturer selling the goods they produce to a distributor or retailer. The public and charitable sectors are also distributors, providing goods, services, and infrastructure, that are in competition with the private businesses, such as dentistry services in the UK. Thus,

$$\boldsymbol{i}_{S} \stackrel{\text{def}}{=} \begin{Bmatrix} \boldsymbol{i}_{\beta}^{S}, \boldsymbol{i}_{B+\beta}^{S} \\ \boldsymbol{i}_{\tau}^{S}, \boldsymbol{i}_{B+\tau}^{S} \\ \boldsymbol{i}_{\phi}^{S}, \boldsymbol{i}_{B+\phi}^{S} \end{Bmatrix}, \;\; \boldsymbol{i}_{2S} \equiv \boldsymbol{i}_{S}. \qquad 42$$

Following the notation convention discussed in Section 4.1.1, then a known subset of suppliers is represented by $\boldsymbol{j}_S$. Also, as the set of retailers $\boldsymbol{j}_{2S}$ and the set of businesses $\boldsymbol{j}_S$ are from the same set, they can be equivalent. Therefore,

$$\boldsymbol{j}_{S} \stackrel{\text{def}}{=} \begin{Bmatrix} \boldsymbol{j}_{\beta}^{S}, \boldsymbol{j}_{B+\beta}^{S} \\ \boldsymbol{j}_{\tau}^{S}, \boldsymbol{j}_{B+\tau}^{S} \\ \boldsymbol{j}_{\phi}^{S}, \boldsymbol{j}_{B+\phi}^{S} \end{Bmatrix}, \;\; \boldsymbol{j}_{S} \subset \boldsymbol{i}_{S}, \;\;\; \boldsymbol{j}_{2S} \equiv \boldsymbol{j}_{S}. \qquad 43$$

A known business supplier group is represented by k_S. This is a supplier belonging to the set of all suppliers $\boldsymbol{i}_S$. This group can only represent one supplier category in their respective economic region,

$$k_{S} \stackrel{\text{def}}{=} \begin{cases} \boldsymbol{j}_{\beta}^{S}, \boldsymbol{j}_{B+\beta}^{S} \\ \boldsymbol{j}_{\tau}^{S}, \boldsymbol{j}_{B+\tau}^{S}, \\ \boldsymbol{j}_{\phi}^{S}, \boldsymbol{j}_{B+\phi}^{S} \end{cases} k_{S} \subset \boldsymbol{i}_{S}, \;\;\; k_{2S} \equiv k_{S}. \qquad 44$$

Not all business groups are producers, but all must be retailers, so we can define the list of producers as,

$$\boldsymbol{i}_{1S} \subset \boldsymbol{i}_{S}, \;\; \boldsymbol{i}_{1S} \subset \boldsymbol{i}_{2S}, \qquad 45$$

a subset of business groups that are producers as,

$$\boldsymbol{j}_{1S} \subset \boldsymbol{i}_{1S}, \;\;\; \boldsymbol{j}_{1S} \not\equiv \boldsymbol{j}_{S}, \qquad 46$$

and a specific business group of producers as,

$$k_{1S} \subset \boldsymbol{i}_{1S}, \;\;\; k_{1S} \not\equiv k_{S}. \qquad 47$$

4.1.1.2.3 Suppliers (labour)

This list of all labour supply (by occupation and experience) is,

$$\boldsymbol{i}_{o} \stackrel{\text{def}}{=} \{\boldsymbol{i}_{\alpha}^{S}, \boldsymbol{i}_{e+}^{S}\}, \qquad 48$$

a subset of the labour supply,

$$\boldsymbol{j}_{o} \stackrel{\text{def}}{=} \{\boldsymbol{j}_{\alpha}^{S}, \boldsymbol{j}_{e+}^{S}\}, \; \boldsymbol{j}_{o} \subset \boldsymbol{i}_{o}, \qquad 49$$

and a specific labour supply subgroup,

$$k_o \stackrel{\text{def}}{=} \{k_\alpha^S, k_{e+}^S\},\ k_o \subset \boldsymbol{i}_o\ . \quad 50$$

4.1.2 RELATIONSHIP INDEX

The difference between a conventional index that identifies the position in a list or array, and a relationship index, is that the latter identifies the position in several inter-related lists or arrays. This method has a huge computational processing saving, benefiting from the approach that the system is stable and financial decisions will follow the previous pattern unless events occur to change the consumer or retailer behaviour.

The supply side relationship index of suppliers is as follows:

$\boldsymbol{j}_{1S_S}$ is the list of producers determined by the supply, Equation 51.
$\boldsymbol{j}_{2S_S}$ is the list of retailers by producers, Equation 52.
$\boldsymbol{j}_{3D_S}$ is the list of consumers by supplier, Equation 53.
$\boldsymbol{j}_{3D_{SB}}$ is the list of consumers that are businesses by supplier, Equation 54.

The demand side relationship index of consumers is as follows:

$\boldsymbol{j}_{3D_D}$ is the list of consumers by supply, Equation 55.
$\boldsymbol{j}_{3D_{DB}}$ is the list of consumers that are businesses by supply, Equation 56.
$\boldsymbol{j}_{2S_D}$ is the list of retailers by consumer, Equation 57.
$\boldsymbol{j}_{2S_{DB}}$ is the list of retailers by business consumer, Equation 58.
$\boldsymbol{j}_{1S_D}$ is the list of producers by retailer by consumer, Equation 59.
$\boldsymbol{j}_{1S_{DB}}$ is the list of producers by retailer by business consumer, Equation 60.
$\boldsymbol{j}_{b_{3D}}$ is the list of input supplies by consumer, Equation 61.
$\boldsymbol{j}_{b_{1S}}$ is the list of input supplies by producer, Equation 64.

The supplies and the retailers by consumer:

$\boldsymbol{j}_{[1S,2S]_{b_{3D}}}$ is the list of supplies and the retailers by all consumers, Equation. 62
$\boldsymbol{j}_{[1S,2S]_{b_{3DB}}}$ is the list of supplies and the retailers by business consumers, Equation 63.
$\boldsymbol{j}_{[3D,2S]_{b_{1S}}}$ is the list of supplies and the retailers by producers, Equation 65.

The list of labour supply, for occupations and experience, by business demand:

$\boldsymbol{j}_{[\alpha o]_{o_{3DB}}}$ is the list of proportion of household subgroup providing labour Equation 66.

These indices are defined in more detail below.

4.1.2.1 Supplier by supply

The producers of each supply in the list $\boldsymbol{i}_b$ are represented by the list $\boldsymbol{j}_{1S_S}$, which are the primary suppliers determined by supply. Not all businesses are producers, as some are distributors. We know which businesses, in which economics regions, produce that supply. So, the index vector $\boldsymbol{j}_{1S_S}$ lists the economic regions and the respective business categories that output each supply in the list $\boldsymbol{i}_b$, which is a subset of all business suppliers $\boldsymbol{j}_S$. Therefore

$$\boldsymbol{j}_{1S_S} \stackrel{\text{def}}{=} [\boldsymbol{j}_S]_{(\boldsymbol{i}_b)}^{(N_b)}. \quad 51$$

As each value in the list of $\boldsymbol{i}_b$ has a new list of values for $\boldsymbol{j}_S$, the resulting index is a matrix of columns (of lists) determined by the length of the list $\boldsymbol{i}_b$, which is N_b. The square brackets $[\dots]$ represent the columns of lists, which as they are different lengths is known a ragged array.

In turn, the list of the producers $\boldsymbol{j}_{1S_S}$ of length N_{1S_S} determines the economic region and the business category of the businesses who retail or distribute that produce. This is a subset of all business suppliers $\boldsymbol{j}_{2S}$, and so there is new list $\boldsymbol{j}_{2S}$ determined by the output supplies (and their producer) they retail and distribute. Thus,

$$\boldsymbol{j}_{2S_S} \stackrel{\text{def}}{=} [\boldsymbol{j}_S]_{(j_{1S_S})}^{(N_{1S_S})}. \quad 52$$

4.1.2.2 Consumer by supplier

As consumer behaviour is stable, unless there is pressure to cause consumer choice to change, then the list of consumers $\boldsymbol{j}_{D_S}$ is determined separately for each retailer output supply in the index $\boldsymbol{j}_{2S_S}$, that is length N_{2S_S}, such that,

$$\boldsymbol{j}_{3D_S} \stackrel{\text{def}}{=} [\boldsymbol{j}_D]_{(j_{2S_S})}^{(N_{2S_S})}. \quad 53$$

In evaluating the cost price of production, then only businesses are the consumers, whereby,

$$\boldsymbol{j}_{DB_S} \stackrel{\text{def}}{=} [\boldsymbol{j}_{DB}]_{(j_{2S_S})}^{(N_{2S_S})}. \quad 54$$

4.1.2.3 Consumer by supply

In the approach defined in this section the consumer is determined by the supply. The consumer can choose to change the supply or the supplier, and this is a decision process which must be stimulated by another event. Once they have selected a supply and retailer, then regular purchases will tend to follow a regular pattern of trade. As consumer behaviour is stable, unless there is pressure to cause consumer choice to change, then the list of consumers is determined by the supply $\boldsymbol{i}_b$ index, so that,

$$\boldsymbol{j}_{3D_D} \stackrel{\text{def}}{=} [\boldsymbol{j}_D]_{(\boldsymbol{i}_b)}^{(N_b)}. \quad 55$$

In evaluating the cost price of production, then only businesses consumer are considered, so that

$$\boldsymbol{j}_{DB_D} \stackrel{\text{def}}{=} [\boldsymbol{j}_{DB}]_{(\boldsymbol{i}_b)}^{(N_b)}. \quad 56$$

4.1.2.4 Supplier by consumer

The list of the consumers $\boldsymbol{j}_{3D_D}$, that is length N_{3D_D}, determines the economic region and the business category of the businesses who retail or distribute that produce,

$$\boldsymbol{j}_{2S_D} \stackrel{\text{def}}{=} [\boldsymbol{j}_S]_{(j_{3D_D})}^{(N_{3D_D})},\ \boldsymbol{j}_{2S_D} \equiv \boldsymbol{j}_{S_D}, \quad 57$$

and for only business consumers it is,

$$\boldsymbol{j}_{2S_{DB}} \stackrel{\text{def}}{=} [\boldsymbol{j}_S]_{(j_{DB_D})}^{(N_{DB_D})},\ \boldsymbol{j}_{2S_{DB}} \equiv \boldsymbol{j}_{1S_{DB}}. \quad 58$$

The list of retailers and distributors $\boldsymbol{j}_{2S_D}$ determines the economic region and the business category of the businesses who are producers,

$$\boldsymbol{j}_{1S_D} \stackrel{\text{def}}{=} [\boldsymbol{j}_S]_{(j_{2S_D})}^{(N_{2S_D})}, \tag{59}$$

and the list of producers delivering business inputs is,

$$\boldsymbol{j}_{1S_{DB}} \stackrel{\text{def}}{=} [\boldsymbol{j}_S]_{(j_{2S_{DB}})}^{(N_{2S_{DB}})}. \tag{60}$$

4.1.2.5 Supply and the retailer by consumer

The business consumer k_{DB} has a list of input supplies $\boldsymbol{j}_{b_{DB}}$, of length $N_{b_{DB}}$, that determine the supplier (the economic region and business category) of each of those supplies k_{2S}. The economic region and retail business the supply is purchased from k_{2S} determines the retail price and taxes on each unit of supply. So, the framework must gather the information on where input supplies are purchased. The list of input supplies by business consumer $\boldsymbol{j}_{b_{DB}}$ is,

$$\boldsymbol{j}_{b_{DB}} \stackrel{\text{def}}{=} \boldsymbol{j}_{b\,(k_{DB})}. \tag{61}$$

This information then determines the unit retail price and the sales taxes. The indexing of the brand and the retailer for all consumers $\boldsymbol{j}_{[1S,2S]_{b_{3D}}}$ is,

$$\boldsymbol{j}_{[1S,2S]_{b_{3D}}} \stackrel{\text{def}}{=} \left\{ [k_{1S} \quad k_{2S}]_{(j_{b_{3D}})}^{(N_{b_{3D}})} \right\}_{(k_{3D})}, \tag{62}$$

or for business consumers $\boldsymbol{j}_{[1S,2S]_{b_{DB}}}$ is,

$$\boldsymbol{j}_{[1S,2S]_{b_{DB}}} \stackrel{\text{def}}{=} \left\{ [k_{1S} \quad k_{2S}]_{(j_{b_{DB}})}^{(N_{b_{DB}})} \right\}_{(k_{DB})}. \tag{63}$$

4.1.2.6 Supply and the retailer by producer

The producer k_{1S} has a list of output supplies $\boldsymbol{j}_{b_{1S}}$, of length $N_{b_{1S}}$, that determine the distributor/retailer (the economic region and business category) of each of those supplies k_{2S}. The economic region and retail business the supply is sold from k_{2S} and where they are purchased k_{3D} determine the retail price and taxes on each unit of supply. So, the framework must gather the information on where input supplies are sold from. The list of input supplies by producer $\boldsymbol{j}_{b_{1S}}$ is,

$$\boldsymbol{j}_{b_{1S}} \stackrel{\text{def}}{=} \boldsymbol{j}_{b\,(k_{1S})}. \tag{64}$$

This information then determines the unit retail price and the sales taxes. The indexing of the brand and the retailer for all consumers $\boldsymbol{j}_{[3D,2S]_{b_{1S}}}$ is,

$$\boldsymbol{j}_{[3D,2S]_{b_{1S}}} \stackrel{\text{def}}{=} \left\{ [k_{3D} \quad k_{2S}]_{(j_{b_{1S}})}^{(N_{b_{1S}})} \right\}_{(k_{1S})}. \tag{65}$$

4.1.2.7 Labour supply by business demand

Like supply and the retailer by consumer, discussed above, the business consumer k_{DB} determines the list of labour (occupation category and level) demanded to produce output supplies. The index value of the business consumer k_{DB} determines the occupations and level, which determines the economic region and the proportion of households in that region supplying the labour. A specific labour supply group k_o representing a category of occupation and a given level of expertise is defined in Equation 50 (Section 4.1.1.2.3) and $\sigma_{\alpha o}$ is the ratio of houses in a subgroup that are fulfilling the labour demand, thus,

$$\boldsymbol{j}_{[\alpha o]_{o_{DB}}} \stackrel{\text{def}}{=} \left\{ [k_o \quad \sigma_{\alpha o}\ ]_{(j_{o_{DB}})}^{(N_{o_{DB}})} \right\}_{(k_{DB})} . \qquad 66$$

4.1.3 VARIABLES

Variables must represent every quantifiable parameter within an economy. A full and detailed list of the Economy Dynamics symbols is provided in *Economy Dynamics Symbols* (see References).

Economy Dynamics uses probabilistic relationships to define the system. This means that for every variable there are two forms: one that represents the value of an individual supply, household, or business and the other represents the probabilistic parameters determined for a group of supplies, households, or businesses. The distinction is represented by using a lowercase letter for the individual supply, household, or business, and by using an uppercase letter for a group of supplies, households or businesses that have similar economic characteristics.

If we consider the cost price of bread then the difference in symbols represents, the cost price of production of an individual independent bakery producing bread p_c and the supply weighted (mean) cost price of production of all the small, independent bakeries producing bread within an economic region P_c.

For some variables, such as demand D and supply S, it is necessary to create accounts that record the totals within an economic region, respectively $\bar{D}$ and $\bar{S}$, and the totals within the simulation (all economies), respectively $\bar{\bar{D}}$ and $\bar{\bar{S}}$.

A summary of the variables used in this paper are provided below:

$\boldsymbol{A}$	is the vector of attributes associated with a value of an index.
c, C	is the non-monetary unit cost of production, including environmental costs (such as greenhouse gas production) and ethical costs (if defined).
d, D	is the consumer demand, the number of units of supply required.
m, M	is the money, such as cash at the bank.
p, P	is the price of a unit of supplies: p_c cost price, and p_r retail price excluding duties and taxes, p_T price of tax, and p_x price paid by customer including duties and taxes.
q, Q	is the value of a quantity: q_A fees for leasing/renting assets, q_D value of fees for debt, q_{p_c} total cost of production, q_Σ value of supplies, and q_T value of tax.
r, R	is the financial rate: r_β business net profit margin, r_D rate of debt interest, and r_T rate of tax.
s, S	is the supply, the number of units of supplies being sold.
t	is the time duration.
$\boldsymbol{w}$	is the surrogate model parameter vector for fitting to historical data.
y, Y	is the income: household income y_α, private business turnover y_β, public business revenue y_τ, and charity donated funds y_ϕ.
z, Z	is the value of asset or debt owned: asset z_A and debt z_D.
σ	is the non-financial ratio: $\sigma_{S_{\Delta s}}$ ratio of unsold supplies surviving to the next timestep and $\sigma_{D_{\Delta d}}$ ratio of unfulfilled demand carrying over to the next timestep.

In understanding the relationship between businesses and consumers we must also distinguish between the supplies available within a catchment region S_S, *supplies-supplied*, the supplies that are sold by businesses S_D, *supplies-demanded*, the supplies a consumer requires within a catchment region D_D, *demand-demanded*, and the supplies consumers buy D_S, *demand-supplied*.

4.1.4 OBJECTS

Given the extent of the equations and the data, it is important to be able to visualise the analytical function and data management structures of Economy Dynamics. In this section the computational objects for Supply and Demand are presented. The computational data structure is based on the relationship between variables, such as the unit cost price being specific to a producer. The relationship of all variables within an economy must be defined to be representative of the observable relationship to enable a robust simulation of the system. In Economy Dynamics any item (tangible or intangible) that can be sold is defined in the supply object and any object that can be bought is defined in the demand object.

The instantiated objects within a simulation do not represent an individual unit of supply but represents a group being treated as a probabilistic model. For example, the instantiated object may represent all bread products in all the bakeries of similar financial characteristics, within an economic region. The bread will be sold to the consumers, which will be distributed as a proportion of the demand of the consumer groups being fulfilled. Using this methodology, we can represent the proportion of households and businesses (such as restaurants) within a subgroup that are unable to buy the bread they require, because it is not available or because they cannot afford it.

4.1.4.1 Supply object

This section defines the data ontology of the supply object $o_{S(i_t)}$, which is updated at every simulation timestep i_t. Equation 67 presents an abridged form of the object with the attribute parameters for each index represented by vector $\boldsymbol{A}$. The structure is then explained and a full representation is provided in Equation 72.

$$o_{S(i_t)} \stackrel{\text{def}}{=} \left[N_b \quad \boldsymbol{i}_b \left\langle \begin{matrix} N_{1S_S} \\ \boldsymbol{A}_{b_S} \end{matrix} \right. \quad \boldsymbol{j}_{1S_S} \left\langle \begin{matrix} N_{2S_S} \\ \boldsymbol{A}_{1S_S} \end{matrix} \right. \quad \boldsymbol{j}_{2S_S} \left\langle \begin{matrix} N_{3D_S} \\ \boldsymbol{A}_{2S_S} \end{matrix} \right. \quad \boldsymbol{j}_{3D_S} \langle \boldsymbol{A}_{3D_S} \right]$$

Equation 67. Abridged supply object

Starting on the left of the object definition, N_b is the number of categories in the index vector $\boldsymbol{i}_b$ next along on the right. The index vector $\boldsymbol{i}_b$ represents all categories of goods, services and assets that can be bought within an economy (see Section 4.1.1.1).

The open angle bracket $\langle$ after $\boldsymbol{i}_b$ indicates that all variables and vectors on the right (N_{1S_S}, $\boldsymbol{A}_{b_S}$, $\boldsymbol{j}_{1S_S}$) are attributes of $\boldsymbol{i}_b$. N_{1S_S} is the number of categories in the index vector $\boldsymbol{j}_{1S_S}$ next along on the right. The index vector $\boldsymbol{j}_{1S_S}$ is the list of different business categories that produce the supply represented by a give index value within vector $\boldsymbol{i}_b$ (see Section4.1.2.1).

This means that for each value of $\boldsymbol{i}_b$ the length N_{1S_S} of $\boldsymbol{j}_{1S_S}$ can change as the list of index values for the respective supply producing business categories will change. For example, the index values in $\boldsymbol{i}_b$ separately representing, say, food, clothing, and furniture. Therefore, each index value will have a different list of business categories producing these goods $\boldsymbol{j}_{1S_S}$, and the length of the list of business N_{1S_S} can be different.

$\boldsymbol{A}_{b_S}$ are the supply-supply attributes of each value for $\boldsymbol{i}_b$,

$$\boldsymbol{A}_{b_S} = \begin{bmatrix} \bar{\bar{S}}_S \\ \bar{\bar{S}}_D \end{bmatrix}, \quad 68$$

where:

$\bar{\bar{S}}_S$	is the total supply available (supplied) within the simulation timestep.
$\bar{\bar{S}}_D$	is the total supply sold (demanded) within the simulation timestep.

Next along, the open angle bracket ⟨ after $\boldsymbol{j}_{1S_S}$ indicates that all variables and vectors on the right (N_{2S_S}, $\boldsymbol{A}_{1S_S}$, $\boldsymbol{j}_{2S_S}$) are attributes of $\boldsymbol{j}_{1S_S}$. N_{2S_S} is the number of categories in the index vector $\boldsymbol{j}_{2S_S}$. The index vector $\boldsymbol{j}_{2S_S}$ is the list of different business categories that retail the supply produced by the business category index value within vector $\boldsymbol{j}_{1S_S}$ (see Section4.1.2.1).

This means that for each value of $\boldsymbol{j}_{1S_S}$ the length N_{2S_S} of $\boldsymbol{j}_{2S_S}$ can change as the list of index values for the respective supply retailing business categories will change. For example, not all retailers stock the same categories of food (such as may be grouped by ethical and sustainable production practices).

$\boldsymbol{A}_{1S_S}$ are the supply attributes of each value for $\boldsymbol{j}_{1S_S}$,

$$\boldsymbol{A}_{1S_S} = \begin{bmatrix} S_{ns} \\ S_{\Delta s} \\ \sigma_{S_{\Delta s}} \\ S_{1s_S} \\ S_{1s_D} \end{bmatrix}, \quad 69$$

where:

S_{ns}	is the volume of new supplies produced in a timestep.
$S_{\Delta s}$	is the volume of unsold supplies from the previous timestep.
$\sigma_{S_{\Delta s}}$	is the proportion of unsold supplies, that have not spoilt, available for purchase.
S_{1s_S}	is the volume of supplies available (on the shelf) in a timestep.
S_{1s_D}	is the volume of supplies demanded (sold) in a timestep.

The third open angle bracket ⟨ after $\boldsymbol{j}_{2S_S}$ indicates that all variables and vectors on the right (N_{3D_S}, $\boldsymbol{A}_{2S_S}$, $\boldsymbol{j}_{3D_S}$) are attributes of $\boldsymbol{j}_{2S_S}$. N_{D_S} is the number of categories in the index vector $\boldsymbol{j}_{3D_S}$. The index vector $\boldsymbol{j}_{3D_S}$ is the list of different consumer categories that demand the supply retailed by the business category index value within vector $\boldsymbol{j}_{2S_S}$ (see Section4.1.2.2).

$\boldsymbol{A}_{2S_S}$ are the supply attributes of each value for $\boldsymbol{j}_{2S_S}$,

$$\boldsymbol{A}_{2S_S} = \begin{bmatrix} C_c \\ P_c \\ P_r \\ \sigma_{ys} \\ \sigma_{2S_S} \\ S_{2s_S} \\ S_{2s_D} \end{bmatrix}, \quad 70$$

where:

C_c	is the unit environmental cost, including distribution costs.
P_c	is the unit cost price of production, including distribution costs.

P_r	is the unit retail price excluding sales taxes.
σ_{yS}	is the unit retail price factor on price, below which increases consumer demand.
σ_{2S_S}	is the ratio of the total supplies retailed, from a given producer.
S_{2s_S}	is the volume of supplies supplied (available).
S_{2s_D}	is the volume of supplies demanded (sold).

The final open angle bracket ⟨ after $\boldsymbol{j}_{3D_S}$ indicates that the vector $\boldsymbol{A}_{3D_S}$ on the right are attributes of $\boldsymbol{j}_{3D_S}$. $\boldsymbol{A}_{3D_S}$ are the supply attributes of each value for $\boldsymbol{j}_{3D_S}$,

$$\boldsymbol{A}_{3D_S} = \begin{bmatrix} S_{3D_S} \\ S_{3D_D} \end{bmatrix}, \qquad 71$$

where:

S_{3D_S}	is the volume of supplies supplied (available).
S_{3D_D}	is the volume of supplies demanded (sold).

Note that supplies supplied by the producer, through the retailer to the consumer is probably not a useful variable. It is quantifying the volume of supplies available to the consumer. It has been included for the aesthetic of the object, whereby supplies supplied (available) and supplies demanded (sold) are stored at each entity level (1st producer, 2nd retailer and 3rd consumer).

A mathematical representation of the supply object structure is shown Equation 72.

$$o_{S(i_t)} \stackrel{\text{def}}{=} \left[N_b \quad \boldsymbol{i}_b \left\langle \begin{matrix} N_{1S_S} \\ \bar{\bar{S}}_S \\ \bar{\bar{S}}_D \end{matrix} \right. \quad \boldsymbol{j}_{1S_S} \left\langle \begin{matrix} N_{2S_S} \\ S_{ns} \\ S_{\Delta s} \\ \sigma_{S_{\Delta s}} \\ S_{1s_S} \\ S_{1s_D} \end{matrix} \right. \quad \boldsymbol{j}_{2S_S} \left\langle \begin{matrix} N_{3D_S} \\ C_c \\ P_c \\ P_r \\ \sigma_{yS} \\ \sigma_{2S_S} \\ S_{2s_S} \\ S_{2s_D} \end{matrix} \right. \quad \boldsymbol{j}_{3D_S} \left\langle \begin{matrix} S_{3D_S} \\ S_{3D_D} \end{matrix} \right. \right]$$

Equation 72. Supplies object

Where:

i_t	is the simulation timestep.
$\stackrel{\text{def}}{=}$	the object definition is:
N_b	is the total number of supply categories; length of vector $\boldsymbol{i}_b$.
$\boldsymbol{i}_b$	is the index vector of goods, services, and asset categories.
$\langle$	for every supply; for every value in vector $\boldsymbol{i}_b$:
N_{1S_S}	is the total number of producer categories; length of vector $\boldsymbol{j}_{1S_S}$.
$\bar{\bar{S}}_S$	is the volume of supplies supplied (available) within the entire simulation.
$\bar{\bar{S}}_D$	is the volume of supplies demanded (sold) within the entire simulation.
$\boldsymbol{j}_{1S_S}$	is the list of producer categories by supply.
$\langle$	for every producer; for every value in vector $\boldsymbol{j}_{1S_S}$:
N_{2S_S}	is the total number of retailer categories; length of vector $\boldsymbol{j}_{2S_S}$.
S_{ns}	is the volume of newly created supplies in the timestep.
$S_{\Delta s}$	is the volume of supplies unsold in previous time step.
$\sigma_{S_{\Delta s}}$	is the ratio of unsold supplies that survive for sale.
S_{1s_S}	is the volume of supplies available (on the shelf) in a timestep.
S_{1s_D}	is the volume of supplies demanded (sold) in a timestep.
$\boldsymbol{j}_{2S_S}$	is the list of retailer categories by supplies and producers.
$\langle$	for every retailer; for every value in vector $\boldsymbol{j}_{2S_S}$:
N_{3D_S}	is the total number of consumer categories; length of vector $\boldsymbol{j}_{D_S}$.
C_c	is the unit environmental and ethical cost price of the supply category.
P_c	is the unit monetary cost price of the supply category.
P_r	is the unit retail price of the supply category, excluding sales taxes.
σ_{yS}	is the unit retail price factor on price, below which increases consumer demand.
σ_{2S_S}	is the ratio of the total supplies retailed, from a given producer.
S_{2s_S}	is the volume of supplies supplied (available).
S_{2s_D}	is the volume of supplies demanded (sold).
$\boldsymbol{j}_{3D_S}$	is the list of consumer categories by supplies and retailers.
$\langle$	for every consumer; for every value in vector $\boldsymbol{j}_{3D_S}$:
S_{3D_S}	is the volume of supplies supplied (available).
S_{3D_D}	is the volume of supplies demanded (sold).

4.1.4.2 Demand object

This section defines the data ontology of the demand object $o_{D(i_t)}$, which is updated at every simulation timestep i_t. Equation 73 presents an abridged form of the object with the attribute parameters for each index represented by vector $\boldsymbol{A}$. The structure is then explained and a full representation is provided in Equation 78.

$$o_{D(i_t)} \stackrel{\text{def}}{=} \left[N_b \quad \boldsymbol{i}_b \left\langle \begin{matrix} N_{3D_D} \\ \boldsymbol{A}_{b_D} \end{matrix} \quad \boldsymbol{j}_{3D_D} \left\langle \begin{matrix} N_{2S_D} \\ \boldsymbol{A}_{3D_D} \end{matrix} \quad \boldsymbol{j}_{2S_D} \left\langle \begin{matrix} N_{1S_D} \\ \boldsymbol{A}_{2S_D} \end{matrix} \quad \boldsymbol{j}_{1S_D} \langle \boldsymbol{A}_{1S_D} \right. \right. \right. \right]$$

Equation 73. Abridged demand object

Starting on the left of the object definition, N_b is the number of categories in the index vector $\boldsymbol{i}_b$ next along on the right. The index vector $\boldsymbol{i}_b$ represents all categories of goods, services and assets that can be bought within an economy (see Section 4.1.1.1.1). This is the same as previously described in supply object.

The open angle bracket ⟨ after $\boldsymbol{i}_b$ indicates that all variables and vectors on the right (N_{D_D}, $\boldsymbol{A}_{b_D}$, $\boldsymbol{j}_{D_D}$) are attributes of $\boldsymbol{i}_b$. N_{3D_D} is the number of categories in the index vector $\boldsymbol{j}_{3D_D}$, which is the list of different consumers demanding the supply represented by a give index value within vector $\boldsymbol{i}_b$ (see Section 4.1.2.3).

This means that for each value of $\boldsymbol{i}_b$ the length N_{3D_D} of $\boldsymbol{j}_{3D_D}$ can change as the list of index values for the consumer demanding the respective supply will change. For example, the index values in $\boldsymbol{i}_b$ separately representing, say, food, clothing, and furniture, also represents the subdivision of this supplies. As consumer groups (differentiated by business type or household socio-economic groups) have different needs and requirements, therefore, the list of consumer categories demanding these goods $\boldsymbol{j}_{3D_D}$, and the length of the list of consumers N_{3D_D} can be different.

$\boldsymbol{A}_{b_D}$ are the supply-demand attributes of each value for $\boldsymbol{i}_b$,

$$\boldsymbol{A}_{b_D} = \begin{bmatrix} \overline{\overline{D}}_D \\ \overline{\overline{D}}_S \end{bmatrix}, \tag{74}$$

where:

$\overline{\overline{D}}_D$	is the total number of units of demand required (demanded) within each timestep.
$\overline{\overline{D}}_S$	is the total number of units of demand supplied (bought) within each timestep.

Next along, the open angle bracket ⟨ after $\boldsymbol{j}_{3D_D}$ indicates that all variables and vectors on the right (N_{2S_D}, $\boldsymbol{A}_{3D_D}$, $\boldsymbol{j}_{2S_D}$) are attributes of $\boldsymbol{j}_{3D_D}$. N_{2S_D} is the number of categories in the index vector $\boldsymbol{j}_{2S_D}$. The index vector $\boldsymbol{j}_{2S_D}$ is the list of different business categories that retail the supply demanded by the consumer category index value within vector $\boldsymbol{j}_{3D_D}$ (see Section 4.1.2.4).

This means that for each value of $\boldsymbol{j}_{3D_D}$ the length N_{2S_D} of $\boldsymbol{j}_{2S_D}$ can change as the list of index values for the respective supply retailing business categories will change; not all consumer groups go to the same retailer groups for a given supply.

$\boldsymbol{A}_{3D_D}$ are the supply attributes of each value for $\boldsymbol{j}_{3D_D}$,

$$\boldsymbol{A}_{3D_D} = \begin{bmatrix} D_{nd} \\ D_{\Delta d} \\ \sigma_{D\,\Delta d} \\ D_{3D_D} \\ D_{3D_S} \end{bmatrix}, \quad 75$$

where:

D_{nd}	is the volume of new demand generated in the current period.
$D_{\Delta d}$	is the volume of unfulfilled demand from the previous period.
$\sigma_{D\,\Delta d}$	is the proportion of unfulfilled demand carried over to the next period.
$D_{3\mathrm{D_D}}$	is the volume of demand demanded (available) in the current period.
$D_{3\mathrm{D_S}}$	is the volume of demand supplied (purchased) in the current period.

The third open angle bracket ⟨ after $\boldsymbol{j}_{2S_D}$ indicates that all variables and vectors on the right (N_{1S_D}, $\boldsymbol{A}_{2S_D}$, $\boldsymbol{j}_{1S_D}$) are attributes of $\boldsymbol{j}_{2S_D}$. N_{1S_D} is the number of categories in the index vector $\boldsymbol{j}_{1S_D}$. The index vector $\boldsymbol{j}_{1S_D}$ is the list of different producer categories from whom the supply is demanded by the retail business category index value within vector $\boldsymbol{j}_{2S_D}$ (see Section 4.1.2.4).

$\boldsymbol{A}_{2S_D}$ are the supply attributes of each value for $\boldsymbol{j}_{2S_D}$,

$$\boldsymbol{A}_{2S_D} = \begin{bmatrix} P_T \\ P_x \\ \sigma_{yD} \\ \sigma_{2S\,D} \\ D_{2s_D} \\ D_{2s_S} \end{bmatrix}, \quad 76$$

where:

P_T	the cost of sales tax for a unit of supply, such as VAT.
P_x	the total price paid by the consumer per unit including retail taxes.
σ_{xD}	the factor sales price threshold above which demand decreases.
$\sigma_{2S\,D}$	the ratio of demand from the consumer category.
D_{2s_D}	the volume of demand demanded from the retailer.
D_{2s_S}	the volume of demand supplied by the retailer.

The final open angle bracket ⟨ after $\boldsymbol{j}_{1S_D}$ indicates that the vector $\boldsymbol{A}_{1S_D}$ on the right are attributes of $\boldsymbol{j}_{1S_D}$. $\boldsymbol{A}_{1S_S}$ are the demand attributes of each value for $\boldsymbol{j}_{1S_D}$,

$$\boldsymbol{A}_{1S_D} = \begin{bmatrix} D_{1S_D} \\ D_{1S_S} \end{bmatrix}, \quad 77$$

where:

D_{1S_D}	the volume of demand demanded (available) from the producer.
D_{1S_S}	the volume of demand supplied (purchased) from the producer.

Note that demand demanded by the consumer, through the retailer from the producer is probably not a useful variable. Like the supplies supplied by the producer, through the retailer to the consumer variable, discussed in Section 4.1.4.1, it has been included for the aesthetic of the object.

A mathematical representation of the demand object structure is shown in Equation 78.

$$o_{D\,(i_t)} \stackrel{\text{def}}{=} \left[N_b \quad \boldsymbol{i}_b \left\langle \begin{matrix} N_{D_D} \\ \bar{\bar{D}}_D \\ \bar{\bar{D}}_S \end{matrix} \quad \boldsymbol{j}_{D_D} \left\langle \begin{matrix} N_{2S_D} \\ D_{nd} \\ D_{\Delta d} \\ \sigma_{D_{\Delta d}} \\ D_{3D_D} \\ D_{3D_S} \end{matrix} \quad \boldsymbol{j}_{2S_D} \left\langle \begin{matrix} N_{1S_D} \\ P_T \\ P_x \\ \sigma_{yD} \\ \sigma_{2S_D} \\ D_{2s_D} \\ D_{2s_S} \end{matrix} \quad \boldsymbol{j}_{1S_D} \left\langle \begin{matrix} D_{1S_D} \\ D_{1S_S} \end{matrix} \right.\right.\right.\right. \right]$$

Equation 78. Demand object

Where:

i_t	is the simulation timestep.
$\stackrel{\text{def}}{=}$	the object definition is:
N_b	is the total number of supply categories; length of vector $\boldsymbol{i}_b$.
$\boldsymbol{i}_b$	is the index vector of goods, services, and asset categories.
$\langle$	for every supply; for every value in vector $\boldsymbol{i}_b$:
N_{D_D}	is the total number of consumer categories; length of vector $\boldsymbol{j}_{D_D}$.
$\bar{\bar{D}}_D$	is the volume of demand demanded within the entire simulation.
$\bar{\bar{D}}_S$	is the volume of demand supplied within the entire simulation.
$\boldsymbol{j}_{3D_D}$	is the list of consumer categories by supplies and retailers.
$\langle$	for every consumer; for every value in vector $\boldsymbol{j}_{3D_D}$:
N_{2S_D}	is the total number of retailer categories; length of vector $\boldsymbol{j}_{2S_D}$.
D_{nd}	is the newly created demand in the timestep.
$D_{\Delta d}$	is the demand unfulfilled in previous time step.
$\sigma_{D\,\Delta d}$	is the ratio of unfulfilled demand that carrier over to next timestep.
$D_{3\mathrm{D_D}}$	is the volume of demand demanded (available) in the current period.
$D_{3\mathrm{D_S}}$	is the volume of demand supplied (purchased) in the current period.
$\boldsymbol{j}_{2S_D}$	is the list of retailer categories by demand from consumer category.
$\langle$	for every retailer; for every value in vector $\boldsymbol{j}_{2S_D}$:
N_{1S_D}	is the total number of producer categories; length of vector $\boldsymbol{j}_{1S_D}$.
P_T	is the unit tax price paid by the consumer category.
P_x	is the unit total price paid by the consumer category.
σ_{xD}	the factor on sales price threshold above which decreases demand.
$\sigma_{2S\,D}$	is the ratio of supplies demanded (sold) from to the consumer category.
D_{2S_D}	is the volume of demand demanded, available to be supplied by the retailer.
D_{2s_S}	the volume of demand supplied by the retailer.
$\boldsymbol{j}_{1S_D}$	is the list of producer categories by demand from retailer category.
$\langle$	for every producer; for every value in vector $\boldsymbol{j}_{1S_D}$:
D_{1S_D}	the volume of demand demanded from the producer.
D_{1S_S}	the volume of demand supplied (purchased) from the producer.

4.2 COST PRICE (STEP 1)

4.2.1 MONETARY COST PRICE

The unit cost price of supplies P_c for a subgroup of business within an economic region, varies between business subgroups, economic region and with time. The discretisation of the system transforms the individual cost price p_c from Equation 4 into,

$$P_{c(\boldsymbol{i}_{2S},i_t)} = \frac{Q_{P_c(\boldsymbol{i}_{2S},i_t)}}{S_{ns(\boldsymbol{i}_{2S},i_t)}}, \quad 79$$

where:

$\boldsymbol{i}_{2S}$	is the list of all retailer subgroups in the simulation, Equation 42.
i_t	is the current timestep of output supply production,
Q_{p_c}	is the operational cost, or net cost of production, Equation 80.
S_{ns}	is the number of units of the new supply produced.

The operational cost Q_{P_c} varies between business subgroup, economic region and with time. In the discretised from, the cost of production for an individual business q_{p_c} from Equation 5, becomes the representative cost for a subgroup of businesses,

$$Q_{P_c(\boldsymbol{i}_{2S},i_t)} = Q_{\Sigma ds_\beta(\boldsymbol{i}_{2S},k_t)} + Q_{Y_{\alpha_L}(\boldsymbol{i}_{2S},i_t)} + Q_{\Sigma ds_A(\boldsymbol{i}_{2S},i_t)} + Q_{T_{\beta_{AL}}(\boldsymbol{i}_{2S},i_t)} + Q_{\Sigma ds_D(\boldsymbol{i}_{2S},i_t)} + Q_{T_{\beta_{CT}}(\boldsymbol{i}_{2S},i_t)}, \quad 80$$

where:

$Q_{\Sigma ds_\beta}$	is the cost of input supplies including duties (demand supplied), Equation 81.
k_t	is a past timestep when input supplies were purchased,
$Q_{y_{\alpha_L}}$	is the payment of wages for labour including employment taxes, Equation 82.
$Q_{\Sigma ds_A}$	is the fee for land usage and other leasing services, Equation 83.
$Q_{T_{\beta_{AL}}}$	is the cost of local business taxes such as Businesses Rates, Equation 84.
$Q_{\Sigma ds_D}$	is the cost of debt repayment, Equation 85.
$Q_{T_{\beta_{CT}}}$	is the tax on business profits, such as corporate taxes, Equation 86.

Definition of the variables that comprise the operational cost are detailed below.

The cost of input supplies $Q_{\Sigma ds_\beta}$ must be established for each business category in each economic region k_{DB} from the list $\boldsymbol{i}_{DB}$. Each input supply is purchased from a business category in an economic region k_{2S} from the list $\boldsymbol{i}_{2S}$. For every business category in k_{DB} there is a list of regular input supplies, and the retailers used for the purchase of those supplies, which is denoted as $\boldsymbol{j}_{[1S,2S]_{b_{DB}}}$. To produce output supplies at the present time i_t, the input supplies must be bought at a previous time k_t. The discretisation of the model transforms $q_{\Sigma ds_\beta}$ from Equation 6 into,

$$Q_{\Sigma ds_\beta(k_{DB},k_t)} = \sum_{\boldsymbol{j}_{[1S,2S]_{b_{DB}}}=1}^{N_{b_{DB}}} D_{b(k_{DB},k_b,k_t)} P_{r(k_b,k_{2S},k_{1S},k_t)} \left(1 + R_{\beta_{T_V}(k_b,k_{2S},k_{DB},k_t)}\right), \quad 81$$

where:

k_{DB}	is a given retailer for an output supply, Equation 41.
k_t	is the past time step when the input supply was purchased.
$\boldsymbol{j}_{[1S,2S]_{b_{DB}}}$	is the input supply category and brand from a retailer, Equation 63.
$N_{b_{DB}}$	is the number of input supply categories.

D_b	is the number of units demanded (purchased).
k_b	is the input supply.
P_r	is the retail price, Equation 95.
k_{2S}	is the retailer, Equation 44.
k_{1S}	is the brand or producer, Equation 47.
$R_{\beta_{T_V}}$	is the fee for land usage and other leasing services.

The total cost of wages $Q_{Y_{\alpha_L}}$ must be established for each business category in each economic region k_{DB} from the list $\boldsymbol{i}_{DB}$. To produce output supplies at the present time i_t in the discretised form then $q_{Y_{\alpha_L}}$ from Equation 7 becomes,

$$Q_{Y_{\alpha_L}(k_{DB},i_t)} = \sum_{\boldsymbol{j}_{[\alpha o]_{o_{DB}}}=1}^{N_{o_{DB}}} D_{L(k_{DB},k_{\alpha o},i_t)} Y_{\alpha_L(k_{\alpha o},i_t)} \left(1 + R_{\beta_{T_Y}(k_{\alpha o},k_{DB},i_t)}\right), \quad 82$$

where:

$\boldsymbol{j}_{[\alpha o]_{o_{DB}}}$	is the occupation category and experience level by business demand, Equation 66.
$N_{o_{DB}}$	is the number of occupation categories, also grouped by experience/seniority.
D_L	is the effort, in terms of the number of wages or salaries consumed in production.
$k_{\alpha o}$	is the economic region the source of labour is demanded from.
Y_{α_L}	is the wage or salary. The rate of the pay for labour will be defined in Paper 07.
$R_{\beta_{T_Y}}$	is the rate of employment tax including both employee and employer taxes.

The total cost of rent and asset leasing $Q_{\Sigma ds_A}$ must be established for each business category in each economic region, which is denoted by the value k_{DB} from the list $\boldsymbol{i}_{DB}$. To produce output supplies at the present time i_t, in the discretised form then $q_{\Sigma ds_A}$ from Equation 8 becomes,

$$Q_{\Sigma ds_A(k_{DB},i_t)} = \sum_{\boldsymbol{j}_{[1S,2S]_{bA_{DB}}}=1}^{N_{j_{bA}}} D_{bA(k_{DB},k_{bA},i_t)} \left(P_{r(k_{bA},k_{2S},k_{1S},i_t)} \left(1 + R_{\beta_{T_V}(k_{bA},k_{2S},k_{DB},i_t)}\right)\right), \quad 83$$

where:

$\boldsymbol{j}_{[1S,2S]_{bA_{DB}}}$	is the input asset category and brand purchased from a given retailer.
$N_{bA_{DB}}$	is the number of input asset categories.
D_{bA}	is the number of units demanded (purchased).
P_r	is the retail leasing price. The value of land and assets will be defined in Paper 06.
$R_{\beta_{T_V}}$	is the tax fee for land usage and other leasing services.

The cost of local business tax $Q_{T_{\beta_{AL}}}$ must be established for each business category in each economic region, which is denoted by the value k_{DB} from the list $\boldsymbol{i}_{DB}$. In the UK, business rates are calculated based on the retail rental value of the business premises $D_{AL}P_{r_{AL}}$, set by the local government. The payment of land usage tax (and other local business taxes) is transformed to the discretised from $q_{T_{\beta_{AL}}}$ of Equation 9 into,

$$Q_{T_{\beta_{AL}}(k_{DB},i_t)} = D_{AL(k_{DB},i_t)} P_{r_{AL}(k_{2S},k_{1S},i_t)} R_{\beta_{T_{AL}}(k_\tau^S,k_{B+}^D,i_t)}, \quad 84$$

where:

D_{AL}	is the number of units of business premises used in operations.

$P_{r_{AL}}$	is the retail unit rental value of the business premise.
k_{2S}	is the retailer.
k_{1S}	is the brand or producer.
$R_{\beta_{T_{AL}}}$	is the business rate of land usage tax.
k_{τ}^{S}	is the economic region determined by local government demarcation.
k_{B+}^{D}	is the category of business paying the tax (for tax banding, if applicable).

In the discretised form, the cost of debt repayment $Q_{\Sigma ds_D}$, like the individual business form $q_{\Sigma ds_D}$ in Equation 10, is a function f_D but distinguishing between economic regions of the supplier and consumer, such that,

$$Q_{\Sigma ds_D(k_{DB},i_t)} = \sum_{\boldsymbol{j}_{[1S,2S]bD_{DB}}=1}^{N_{bD}} f_{D(j_{bD})}\left(-Z_{D(k_{DB},k_{bD})}, R_{D\left(k_{bD},k_{2S},k_{DB},k_{t_{F0}}\right)}, t_{D(j_{bD})}\right), \quad 85$$

where:

$\boldsymbol{j}_{[1S,2S]_{b_{DB}}}$	is the debt supply category and brand purchased from a given retailer.
N_{bD}	is the number of borrowing categories.
$f_D(\ \ )$	is a function of $-Z_D, R_D, t_D, k_{t_{F0}}$, like Equation 11 with a loan agreement date.
Z_D	is the money owed.
k_{bD}	is the loan type.
R_D	is the annual interest rate on the debt.
k_{2S}	is the retailer.
$k_{t_{F0}}$	is the date of the loan agreement.
t_D	is the duration of the repayment schedule.

The functions $f_{D(j_{b_D})}$ are well-defined methods of calculating the repayment schedule. An example is provided in Equation 11 and others are discussed in Maybury 2020 [Paper 01, §6.2.1].

The cost of corporate tax $Q_{T_{\beta_{CT}}}$ is,

$$Q_{T_{\beta_{CT}}(k_{DB},i_t)} = \left(Q_{p_r(k_{DB},i_t)} - Q_{p_c(k_{DB},i_t)}\right) R_{\beta_{T_{CT}}(k_{\tau}^{S},k_{B+}^{D},i_t)}\,, \quad 86$$

where:

Q_{p_r}	is the total sales revenue from the sale of goods and services, Equation 93.
Q_{p_c}	is the total production cost, Equation 80.
$R_{\beta_{T_{CT}}}$	is the rate of corporate tax.
k_{τ}^{S}	is the economic region determined by local government demarcation.
k_{B+}^{D}	is the category of business paying the tax (for tax banding, if applicable).

4.2.2 NON-MONETARY COST PRICE

The calculation of the environment and ethical cost is supplementary to the methods needed for the Economy Dynamics framework to generate repeatable predictions of economic characteristics, such as inflation. The methodology for calculating the data exploits the methodology proposed for cost price of production. The purpose of the calculation is to study the effect of economic regions and social divisions in promoting environmentally unsustainable and ethically unacceptable business practices.

The approach requires the quantification of the environmental and ethical costs in non-monetary terms, such as tonnes of greenhouse gases (GHG) produced per unit supply, C_{GHG}. Additional environmental and ethical cost for different metrics could be included, such as tonnes of plastic waste, hectares of crop monoculture and person months of child labour. The calculation of these values is primarily a matter of accounting; therefore, the quantification of past processes will remain applicable to future analyses. This also creates a long-term feedback loop, enabling the ability to study diminishing production caused by unsustainable practices.

Using GHG as the example, then the calculation is conventionally divided into three scopes:

- **Scope 1**: The production of GHG emissions by a company through fuel consumption, including heating and cooling, and running equipment and vehicles, use of refrigerant systems, and emissions from waste.
- **Scope 2**: The production of GHG emissions by a company through the indirect purchase of energy produced by a supplier, such as electricity consumption including lighting and equipment.
- **Scope 3**: The production of GHG emissions by a company through the business operations, including those of all the input goods, the emissions of employees commuting to work and the cloud computation energy costs, including remote working, and the emission of land usage and depreciable asset production.

Generally, Scope 3 is the largest contributor to GHG emission by companies. Unfortunately, currently, without a framework such as Economy Dynamics, it is hardest to quantify. In contrast, by representing the calculation of GHG emissions as a discretised function, then the calculation is simplified as all input supplies were output supplies produced in the past i_t, in an economic region i_β^D, and were supplied by a given business category i_{B+}^D.

The unit ethical and environmental cost price of supplies, demonstrated by using the example of Green House Gas (GHG) emissions C_{GHG} is,

$$C_{GHG(\boldsymbol{i}_{2S},i_t)} = \frac{Q_{GHG(\boldsymbol{i}_{2S},i_t)}}{S_{ns(\boldsymbol{i}_{2S},i_t)}}, \quad 87$$

where:

$\boldsymbol{i}_{2S}$	is the list of all retailer subgroups in the simulation, Equation 42.
i_t	is the current time step, during production,
Q_{GHG}	is the GHG emissions of total production, Equation 88.
S_{ns}	is the number of new supply units produced.

The non-monetary operational cost Q_{GHG} is determined as the cost of input supplies $Q_{\Sigma d_{GHG}}$, the cost of labour $Q_{L_{GHG}}$, which includes the use of communications infrastructure and services paid for by the employee, and, the cost of assets $Q_{A_{GHG}}$, so that,

$$Q_{GHG(\boldsymbol{i}_{2S},i_t)} = Q_{\Sigma ds_{GHG}(\boldsymbol{i}_{2S},i_t)} + Q_{L_{GHG}(\boldsymbol{i}_{2S},i_t)} + Q_{A_{GHG}(\boldsymbol{i}_{2S},i_t)}, \quad 88$$

where:

$Q_{\Sigma ds_{GHG}}$	is the non-monetary cost of input supplies, inc. fuel and electricity, Equation 89.
$Q_{L_{GHG}}$	is the non-monetary cost of commuting and remote working, Equation 90.
$Q_{A_{GHG}}$	is the non-monetary cost of land and asset usage, Equation 91.

The GHG emission of the output supplies that are purchased as input supplies by other businesses must be accounted for. As the solution for non-monetary cost is resolved iteratively, whereby the effects of variation in workforce and asset usage on the non-monetary cost is captured within the output supplies and these are then purchased as input supplies by other businesses. So, the input supplies equation need only reflect the appropriate discretisation of time, economic region, and business category.

The non-monetary cost of input supplies $Q_{\Sigma d_{GHG}}$ (represented by the GHG variable) is adapted from the monetary cost $Q_{\Sigma ds_{\beta}}$ in Equation 81,

$$Q_{\Sigma ds_{GHG}(k_{DB},k_t)} = \sum_{\boldsymbol{j}_{[1S,2S]_{b_{DB}}}=1}^{N_{b_{DB}}} D_{b(k_{DB},k_b,k_t)} C_{GHG(k_b,k_{2S},k_{1S},k_t)} \,, \quad 89$$

where:

k_{DB}	is a given business consumer requiring an input supply.
k_t	is the past time step when the input supply was purchased.
$\boldsymbol{j}_{[1S,2S]_{b_{DB}}}$	is the input supply category and brand purchased from a given retailer.
$N_{b_{DB}}$	is the number of input supply categories.
D_b	is the number of units demanded (purchased).
k_b	is the input supply.
C_{GHG}	is the non-monetary (GHG) unit cost of production.
k_{2S}	is the retailer.
k_{1S}	is the brand or producer.

The non-monetary cost of the workforce $Q_{L_{\mathrm{GHG}}}$ is,

$$Q_{L_{GHG}(k_{DB},i_t)} = \sum_{\boldsymbol{j}_{[\alpha o]_{o_{DB}}}=1}^{N_{o_{DB}}} D_{L(k_{DB},k_{\alpha o},i_t)} C_{L_{GHG}(k_{\alpha o},i_t)} \,, \quad 90$$

where:

$\boldsymbol{j}_{[\alpha o]_{o_{DB}}}$	is the occupation category including seniority and experience by business demand.
$N_{o_{DB}}$	is the number of occupation categories, including seniority.
D_L	is the effort consumed in production.
$k_{\alpha o}$	is the economic region the source of labour is demanded from.
$C_{L_{GHG}}(j_s)$	is the unit cost, including commuting and remote working overheads not paid for by the company.

The non-monetary cost for the usage of land and assets $Q_{A_{GHG}}$ is,

$$Q_{A_{GHG}(k_{DB},i_t)} = \sum_{\boldsymbol{j}_{[1S,2S]_{bA_{DB}}}=1}^{N_{j_{bA}}} D_{bA(k_{DB},k_{bA},i_t)} C_{A_{GHG}(k_{bA},k_{2S},k_{1S},i_t)} \,, \quad 91$$

where:

$\boldsymbol{j}_{[1S,2S]_{bA_{DB}}}$	is the input asset category and brand purchased from a given retailer,
$N_{bA_{DB}}$	is the number of input asset categories,
D_{bA}	is the given area and/or the number of asset units used,
$C_{A_{GHG}}$	is the unit non-monetary cost price per unit of asset used or leased.
k_{bA}	is the input supply (asset).
k_{2S}	is the retailer.
k_{1S}	is the brand or producer.

4.3 *SET THE RETAIL PRICE* (STEP 2)

4.3.1 CALCULATE PROFIT MARGIN

The methodology of calculating the profit margin R_β for the discretised methodology is the same as the individual methodology r_β, discussed in Section 3.3, except for differentiating by business category $\boldsymbol{i}_{2S}$ and their respective output supply/supplies $\boldsymbol{j}_b$ and the methodology used to estimate the profit margin $\tilde{R}_\beta$ in the simulation.

Therefore, the measured business profit margin R_β is transformed from r_β in Equation 15 into,

$$R_{\beta(\boldsymbol{i}_{2S},i_t)} = \frac{Q_{pr(i_{2S},i_t)} - Q_{pc(i_{2S},i_t)}}{Q_{pr(i_{2S},i_t)}}, \quad 92$$

where:

Q_{p_r}	is the total sales revenue from the sale of goods and services, Equation 93.
Q_{p_c}	is the total production cost, Equation 4.

The total sales revenue Q_{P_r} discretised from q_{P_r} in Equation 16 becomes,

$$Q_{P_r(\boldsymbol{i}_{2S},i_t)} = \sum_{k_b=1}^{N_b} P_{r(\boldsymbol{i}_{2S},k_b,i_t)} S_{D(\boldsymbol{i}_{2S},k_b,i_t)}, \quad 93$$

where:

k_b	is a given output supply category from the list produced $\boldsymbol{j}_b$.
P_r	is the retail price per unit of supply, Equation 95.
S_D	is the number of units of supply sold (demanded).

The discretised equation for the business income y_β (see business Equation 27) from the sale of output supplies $q_{\Sigma sd}$ (see business Equation 28) will be discussed in detail in Paper 02, Addendum A (in progress). The discretised equation for the distribution of profits q_{r_β} (see business Equation 29), which is a constraint on the business viability, such as the need to provide a business owner-worker with sufficient income to live on, will also be discussed in detail in Paper 02, Addendum A (in progress). So, the viability of the business is linked to the ability of any owner-worker households to afford their living costs through the Macroeconomic Primitive (Maybury, 2020 [Paper 02]).

In contrast to the individual business methodology in Equation 19, the discretised methodology estimates profit margin from business data by using a surrogate model[5]. Profit margin of business categories within an economic region can be obtained by training a surrogate model against population datasets. The business demographics surrogate model is represented as function $f_{R_\beta}(\ \ )$. Within the brackets are the list of indices that define the variables and dimensions for the pattern fitting model of the surrogate function. The surrogate model parameter vector $\boldsymbol{w}$ is determined by training or fitting the model process against or to historical population data. Once trained, the

[5] A surrogate model is a mathematical function or computer algorithm used to predict the output of a system given a set of inputs, but with no information about the nature of the system. All surrogate models have parameter values populated by training (machine learning) or fitting (statistical correlation) the model to existing data, such as historical economic and financial data. Once fitted the model can 'predict' the response of the group or population, within the range of input variable values within the historical dataset. Thus, the surrogate model must only be interpolation and not extrapolation.

surrogate model will provide a single value for each unique set of index values by interrogating the surrogate model using the parameter vector $\boldsymbol{w}$.

A surrogate model is generally independent of time. It provides a simulated value of the output, in this case profit margin $\bar{R}_\beta$, based on the value of the input parameters,

$$\bar{R}_{\beta(i_{2S},i_t)} = f_{R_\beta}(k_{2S}, \boldsymbol{w}), \tag{94}$$

where:

k_{2S}	is the business category to provide an estimated profit margin for,
$\boldsymbol{w}$	is the surrogate model trained parameter vector.

To vary the value of the retail price, for a given cost price, is a business decision to change the profit margin based on feedback from the system, such as failing to sell stock or, the opposite, selling out too quickly. The business decision making will be discussed in Paper 05 (in progress) and the feedback is discussed in the next section.

4.3.2 Estimate the Retail Price

The unit retail price of supplies P_r varies between economic regions, business category, and with time. The individual business form of retail price p_r in Equation 2 is transformed into,

$$P_{r(\boldsymbol{i}_{2S},\boldsymbol{i}_t)} = P_{c(\boldsymbol{i}_{2S},\boldsymbol{i}_t)}\left(1 + R_{S(\boldsymbol{i}_{2S},\boldsymbol{i}_t)}\right), \tag{95}$$

where:

$\boldsymbol{i}_{2S}$	is the list of all businesses (retailers), Equation 42.
i_t	is the simulation timestep.
P_c	is the unit cost price of production for a supply category, Equation 79.
R_S	is the estimated sales mark-up price, Equation 96.

The sales-markup factor can be estimated from the net profit margin. By rearranging Equation 3 in terms of the mark-up factor, and discretising the variables, then the relationship between the sales mark-up factor R_S and the net business profit margin, assuming all units produced are sold, is,

$$R_{S(\boldsymbol{i}_{2S},\boldsymbol{i}_t)} = \frac{1}{1-\bar{R}_{\beta(i_{2S},i_t)}} - 1, \tag{96}$$

where:

$\bar{R}_\beta$	is the estimated business profit margin from historical data, Equation 94.

4.3.3 Calculate Gross Price

The gross price P_x, the price the customer pays, includes any retailer duties or taxes based on the supply and the economic region of the seller, and any consumer duties or taxes based on the supply and the economic region of the purchaser. The discretised form is therefore not like the individual form p_x in Equation 21, as it must account for both the region of the seller and the buyer. Therefore,

$$P_{x(k_{2S},k_D,\boldsymbol{i}_t)} = P_{r(k_{2S},\boldsymbol{i}_t)}\left(1 + R_{T_S(k_{2S},\boldsymbol{i}_t)}\right)\left(1 + R_{T_D(k_D,\boldsymbol{i}_t)}\right), \tag{97}$$

where:

k_{2S}	is a given retailer for an output supply, Equation 44.
k_D	is a given business consumer requiring an input supply.
i_t	is the simulation timestep.

P_r	is the retail price per unit of supply, Equation 95.
R_{T_S}	is the duty on the supply coming from the economic region of the supplier.
R_{T_D}	is the duty on the supply applied by to the economic region of the consumer.

The cost of tax P_T, in the discretised form is not like the individual form p_T in Equation 22, owing to the change in P_x. Thus, the function is simply,

$$P_{T(k_{2S},k_D,\boldsymbol{i}_t)} = P_{x(k_{2S},k_D,\boldsymbol{i}_t)} - P_{r(k_{2S},\boldsymbol{i}_t)}\,, \quad 98$$

where:

k_{2S}	is a given retailer for an output supply, Equation 44.
k_D	is a given business consumer requiring an input supply.
i_t	is the simulation timestep.
P_x	is the gross price per unit of supply, Equation 97.
P_r	is the retail price per unit of supply, Equation 95.

4.4 *COMPETITOR PRICE* (STEP 3)

When the business competes directly on price, and price alone, by discretising p_x of Equation 23, then the gross price P_x should be,

$$P_{x(k_{2S},k_D,i_t)} \leq min\left(P_{x(\boldsymbol{j}_{2S},k_D.k_t)}\right), \quad 99$$

where:

k_{2S}	is the retailer subgroup assessing their retail price, Equation 44.
k_D	is a given business consumer requiring an input supply.
$\boldsymbol{j}_{2S}$	is the list of all direct competitor, retailer subgroups, Equation 43.
i_t	is the current timestep of output supply production.
k_t	is the previous timestep of output supply production.

If the inequality in Equation 99 is false, then ideally the retail price must be adjusted so that the inequality becomes true, i.e., $P_{\mathrm{x}(k_{2S},k_{\mathrm{D}},i_t)} = min\left(P_{\mathrm{x}(\boldsymbol{j}_{2S},k_{\mathrm{D}}.k_t)}\right)$.

When the business has brand loyalty amongst its customer, then discretising Equation 24, the gross price must be,

$$P_{r(k_{2S},k_D,i_t)} \leq min\left(P_{r(\boldsymbol{j}_{2S},k_D,k_t)} + \partial P_{r(\boldsymbol{j}_{2S},k_D,k_t)}\right), \quad 100$$

where:

∂	is the price tolerance of existing customers.

If the inequality in Equation 100 is false, then the retail price must be adjusted so that the inequality becomes true, i.e., $P_{r(k_{2S},k_{\mathrm{D}},i_t)} = min\left(P_{r(\boldsymbol{j}_{2S},k_{\mathrm{D}},k_t)} + \partial P_{r(\boldsymbol{j}_{2S},k_{\mathrm{D}},k_t)}\right)$.

For businesses that do not have a dependency on their output supply retail price imposed by the retail price of competitors, it is said to be independent $\perp$ (orthogonal) of the other retail prices in the market, discretising Equation 25, such that,

$$P_{x(k_{2S},k_D,i_t)} \perp P_{x(j_{2S},k_D,k_t)} \,. \quad 101$$

As discussed in Section 3.5 these equations explain why there is a tendency for mature markets to consolidate to only a few major players. Namely, businesses for which Equation 101 is applicable determine the viability of the businesses for which Equations 99 or 100 are applicable. The discretisation between business type and economic region could also explain why markets consolidate to a few national or multinational companies, whereby there is seemingly greater consumer trust in larger organisations. Naturally, this hypothesis requires validating.

4.5 *BUSINESS NEEDS* (STEP 4)

As discussed in Section 2.1.1.1, in relation to owner-workers, businesses have a need for profits Q_{R_β}, and they must be allocated according to that need. The allocation of business profits, by discretising q_{R_β} from Equation 29, then,

$$Q_{R_\beta(\boldsymbol{i}_{2S},i_t)} = Y_{\alpha_B(\boldsymbol{i}_{2S},i_t)} + Y_{\beta_B(\boldsymbol{i}_{2S},i_t)} + M_{\lambda(\boldsymbol{i}_{2S},i_t)} + Q_{\Sigma d(\boldsymbol{i}_{2S},i_t)} \,, \quad 102$$

where:

$\boldsymbol{i}_{2S}$	is the list of all businesses (retailers), Equation 42.
i_t	is the simulation timestep.
Y_{α_B}	is the payment of dividends to the household owners of the business.
Y_{β_B}	is the payment of dividends to the business owners of the business.
M_λ	is the money retained by the business for cash-flow management.
$Q_{\Sigma d}$	is the money reinvestment to maintain and grow the business (one-off purchases).

To estimate the minimum profit, it will be necessary to study the various business categories with a specific view to understanding the constraints on profit margin. For example, in the UK, the profits of a business classed as sole traders will need to be sufficient to meet the living cost of the business worker-owner. The costs of running a household are discussed in Maybury, 2020 [Paper 01].

The surrogate modelling approach is discussed in the foot note of Section 4.3.1. A surrogate model is generally independent of time, so the output may need to be normalised by a typical wage, such as the household economic subgroup running cost, including the Minimum Income Standard (Bradshaw et al., 2008 and Hill et al., 2016).

This will provide a simulated value of the output, in this case minimum profit $\bar{Q}_{\mathrm{R}_\beta}$, based on the value of the input parameters,

$$min\left(\bar{Q}_{R_\beta(\boldsymbol{i}_{2S},i_t)}\right) = f_{Q_{R_\beta}}(k_{2S}, \boldsymbol{w}), \quad 103$$

where:

k_{2S}	is the business category to provide a minimum profit for,
$\boldsymbol{w}$	is the surrogate model trained parameter vector.

The profit based on the competitor constrained retail price must be,

$$Q_{R_\beta(\boldsymbol{i}_{2S},i_t)} \geq min\left(\bar{Q}_{R_\beta(\boldsymbol{i}_{2S},i_t)}\right), \quad 104$$

If the inequality in Equation 104 is false, then the retail price must be adjusted so that the inequality becomes true, i.e., $Q_{\mathrm{R}_\beta(\boldsymbol{i}_{2S},i_t)} = \min\left(\bar{Q}_{\mathrm{R}_\beta(\boldsymbol{i}_{2S},i_t)}\right)$.

If a business is unable to make sufficient profits to ensure viability, and if this situation is prolonged, the business will fail. This topic will be detailed in Paper 05 (Maybury, in progress).

4.6 *Relative price stability* (Step 5)

There is stability in the system owing to the regular financial behaviour of households and businesses being consistent with previous behaviour (Section 2.1.3). However, price and availability affect consumer behaviour. Availability will be considered in Section 4.8. This section evaluates if there are price changes within the system that require calculations to resolve the new consumer behaviour, or if the system is stable and simply repeats the previous timestep.

There are three aspects to price that must be investigated:

1. Has the price of the regular supply changed above a threshold?
2. Has the price of an alternative supply changed below a threshold?
3. Has the relative price of the supplies remained unchanged?

These are discussed in the sections below.

4.6.1 Testing the price of the regular supply relative to a threshold

Testing whether the price is above the threshold of changing consumer behaviour can be represented as an inequality that determines the list of retailers whose price exceeds the consumer threshold $\boldsymbol{j}_{x\mathrm{S}}$. Thus, each business subgroup compares the new retail price to that of the previous timestep.If the price inequality is met, i.e., the price changes consumer behaviour to seek lower cost alternatives, then the retailer index must be stored, else the index can be replaced by a null value (0), such that,

$$P_{r(\boldsymbol{j}_{2S},i_t)} > P_{r(\boldsymbol{j}_{2S},k_t)}(1+\sigma_{xD}),\ \boldsymbol{j}_{xD} = \boldsymbol{j}_{2S}\ ;\ \boldsymbol{j}_{xD} = 0\,, \quad 105$$

where:

σ_{xD}	is the price tolerance factor of existing customers.
$\boldsymbol{j}_{2S}$	is the list of all competitor retailer subgroups in a market, Equation 43.
$\boldsymbol{j}_{xD}$	is the list of all retailers whose retail price exceeds the upper customer threshold.
i_t	is the current timestep of output supply production.
k_t	is the previous timestep of output supply production.

4.6.2 Testing the price of an alternative supply relative to a threshold

Similarly, testing whether the price is below the threshold of changing consumer behaviour can be represented as an inequality that determines the list of retailers whose price is below the consumer threshold $\boldsymbol{j}_{y\mathrm{S}}$. As before, each business subgroup compares the new retail price to that of the previous timestep. If the price inequality is met, i.e., the price changes consumer behaviour to buy the alternative brand or product, then the retailer index must be stored, else the index can be replaced by a null value (0), such that,

$$P_{r(\boldsymbol{j}_{2S},i_t)} < P_{r(\boldsymbol{j}_{2S},k_t)}(1-\sigma_{yS}),\ \boldsymbol{j}_{yS} = \boldsymbol{j}_{2S}\ ;\ \boldsymbol{j}_{yS} = 0\,, \quad 106$$

where:

$\sigma_{y\mathrm{S}}$	is the price tolerance factor of new customers switching supplies.
$\boldsymbol{j}_{2S}$	is the list of all competitor retailer subgroups in a market, Equation 43.

$\boldsymbol{j}_{yS}$	is the list of all retailers whose retail price is below the lower customer threshold.
i_t	is the current timestep of output supply production.
k_t	is the previous timestep of output supply production.

Consumers that purchase from the list of supplies whose prices have changed above the threshold $\boldsymbol{j}_{xS}$ (Equation 105) will know that the prices have changed and to look for an alternative supplier or supply. However, there may be a delay to exploring the alternative options, depending on the affluence of the business and household economic group.

Consumers that would use an alternative supplier or supply, given a sufficiently favourable price from suppliers $\boldsymbol{j}_{yS}$ (Equation 106), will need to be informed of this price change. Whatever the mode by which this information is received, the effect on consumer behaviour can be repressed as a probability distribution with time of consumers switching supplier. This will be discussed in Section 4.7.

4.6.3 Testing the relative price of supplies

To determine if the relative price of supplies has changed, we can test if the relative order, from lowest to highest, of the retail price of all businesses selling competing goods or services has changed. This is achieved by comparing the current ranking of supplier prices with the ranking of the previous timestep, such that,

$$sort\left(\boldsymbol{j}_{2S(i_t)}, P_{r(j_{2S},i_t)}\right) \equiv sort\left(\boldsymbol{j}_{2S(k_t)}, P_{r(j_{2S},k_t)}\right) , \qquad 107$$

where:

$\boldsymbol{j}_{2S}$	is the list of all competitor retailer subgroups in order of price, Equation 43.
i_t	is the current timestep of output supply production.
k_t	is the previous timestep of output supply production.

If the order of supply-suppliers by price remains unchanged (i.e., identical), then consumer behaviour models against price do not require evaluating. In contrast, if the price order has changed, then consumers affected by the lists of $\boldsymbol{j}_{xD}$ and $\boldsymbol{j}_{yS}$ must be evaluated.

4.7 *Consumer behaviour* (*Step 6*)

4.7.1 Demand demanded

Demand demanded is the volume of supplies that customers are seeking to purchase within the current timestep. In Step 6 of the analysis process, the regular consumer behaviour must be adjusted for the effects of price, and this can be performed by using a surrogate model as discussed in Section 2.1.3.3. The options for the change in behaviour from Section 2.1.4 are:

- Change the brand, or select an alternative product, within the same store.
- Change the store but keep the brand at a better price.
- Change the store and change brand such as choosing the own brand of a new store.
- Select a product alternative, such as rice instead of pasta.
- Go without, which needs to be evaluated by consumer financial forcing ([Paper 01]).

This consumer behaviour, in response to an increase in price of the regular supply (retailer and producer), or a decrease in price of the alternative supply (supply type, or retailer and producer), can be represented as surrogate modelling function $f_{D_{\Delta P_x}}(\quad)$. Within the brackets of the function are the list of indices that define the variables and dimensions for the pattern fitting model. The model parameter vector $\boldsymbol{w}$ is determined by a training process against the population data. Once trained, the surrogate model will provide an *interpolated* predicted response for each unique set of index values by interrogating the value surrogate model, for a set of input condition, by using the function populated by the parameter vector $\boldsymbol{w}$. The output from the function will allocate the change in purchasing behaviour, identifying the supply items swapped because of price.

The surrogate model must output a revised demand object (see Equation 73 and Equation 78), updating the index relationship of the supplier and producer by the consumer $\boldsymbol{j}_{[1S,2S]_{b_D}}$. The list of consumers $\boldsymbol{j}_{3D_D}$ demanding a supply $\boldsymbol{i}_b$, may change if the new consumer behaviour is to select an alternative supply, such as using rice instead of pasta. The total number of consumer categories N_{3D_D} must be updated. The vector $\boldsymbol{A}_{3D_D}$ (Equation 75) will need to have the value of new demand D_{nd} updated to reflect any change in the size of the consumer subgroup and the vector could become an attribute of a different supply $\boldsymbol{i}_b$.

The vector of the retailers $\boldsymbol{j}_{2S_D}$ may change if the consumer changes supplier. The total number of retailer categories N_{2S_D} must be updated. The vector $\boldsymbol{A}_{2S_D}$ (Equation 76) must have its values updated for volume of demand from the retailer D_{2S_D}, and the ratio of demand from the consumer category σ_{2S_D}. The ratio of demand from a consumer group for a given retailer is,

$$\sigma_{2S_D}(j_{3D_D}, i_t) = \frac{D_{2S_D}(j_{3D_D}, i_t)}{D_{3D_D}(i_b, i_t)}, \tag{108}$$

where:

$\boldsymbol{j}_{3D_D}$	is the list of consumers by supply, Equation 55.
$\boldsymbol{i}_b$	is a list of supply categories, Equation 35.
$\boldsymbol{i}_t$	is the list of all timesteps.
D_{3D_D}	is the volume of demand from a consumer group, Equation 109.
D_{2S_D}	is the volume of demand from a retailer group, see Equation 111.

The total demand from a consumer subgroup is represented as,

$$D_{3D_D(\boldsymbol{i}_D, i_t)} = D_{nd(\boldsymbol{i}_D, i_t)} + \sigma_{D_{\Delta d}(\boldsymbol{i}_D, i_t)} \Delta D_{3D_D(\boldsymbol{i}_D, i_t)}, \tag{109}$$

where:

$\boldsymbol{i}_D$	list of all consumer groups, including household and businesses, Equation 36.
$\boldsymbol{i}_t$	is the list of all timesteps.
D_{nd}	is the volume of new demand generated in the current period.
$\sigma_{D_{\Delta d}}$	is the proportion of unfulfilled demand carried over to the next period.
ΔD_{3D_D}	is the volume of unfulfilled demand from the previous period.

The vector of the producers $\boldsymbol{j}_{1S_D}$ may change if the brand of the consumer demand changes. The total number of producer categories N_{1S_D} must be updated. The vector $\boldsymbol{A}_{1S_D}$ (Equation 77) must have its values updated for the volume of demand supplied by the producer D_{1S_S}.

4.7.2 Surrogate Model

The updates to the demand object $o_{D(i_t)}$ by the surrogate function $f_{D_{\Delta P_x}}(\ )$ is summarised in Equation 110, below.

$$o_{D(i_t)}\left[N_{3D_D},\boldsymbol{j}_{3D_D},N_{2S_D},D_{nd},\boldsymbol{j}_{2S_D},N_{1S_D},D_{2S_D},\sigma_{2S_D},\boldsymbol{j}_{1S_D},D_{1S_S}\right]=f_{D_{\Delta P_x}}\left(\boldsymbol{j}_{xS},\boldsymbol{j}_{yS},\boldsymbol{j}_{[1S,2S]_{b_{3D}}(k_t)}\right), \quad 110$$

where:

i_t	is the current timestep of output supply production.
N_{3D_D}	is the total number of consumer categories; length of vector $\boldsymbol{j}_{D_D}$.
$\boldsymbol{j}_{3D_D}$	is the list of consumers by supply, Equation 55.
N_{2S_D}	is the total number of retailer categories; length of vector $\boldsymbol{j}_{2S_D}$.
D_{nd}	is the newly created demand in the timestep.
$\boldsymbol{j}_{2S_D}$	is the list of retailers by consumer, Equation 57.
N_{1S_D}	is the total number of producer categories; length of vector $\boldsymbol{j}_{1S_D}$.
D_{2S_D}	is the demand available (to be bought) from the retailer.
σ_{2S_D}	is the ratio of supplies demanded by the consumer category.
$\boldsymbol{j}_{1S_D}$	is the list of producers by retailer by consumer, Equation 59.
D_{1S_S}	is the demand supplied by the producer.
$\boldsymbol{j}_{xS}$	is the list of retailers whose retail price exceeds the upper customer threshold, Equation 105.
$\boldsymbol{j}_{yS}$	is the list of retailers whose retail price is below the lower customer threshold, Equation 106.
$\boldsymbol{j}_{[1S,2S]_{b_{3D}}}$	is the list of supplies and the retailers by all consumers, Equation. 62.
k_t	is the previous timestep of output supply production.

The characteristics of the surrogate model must include the probability of consumers becoming aware of an alternative supply dropping the price and then switching producer (brand) or retailer (outlet) accordingly and how the update is modelled over time. It must also ensure that consumers are within the catchment area of the supply being switched to.

The matter of affordability requires the knowledge of the consumer income/turnover and cost-of-living/running costs, so affordability decisions must be handled by the financial forcing function for households (Maybury 2020, [Paper 01]) and the financial forcing function for businesses ([Paper 05], in progress).

4.7.3 Relationship between the levels

The relationship between the demand variables in each of the demand vectors in the demand object is represented as,

$$\overline{\overline{D}}_{D(k_b,\boldsymbol{i}_t)}=D_{3D_D\left(\boldsymbol{j}_{3D_D},\boldsymbol{i}_t\right)}=D_{2S_D\left(\boldsymbol{j}_{2S_D},\boldsymbol{i}_t\right)}=D_{1S_D\left(\boldsymbol{j}_{1S_D},\boldsymbol{i}_t\right)}, \quad 111$$

where:

$\overline{\overline{D}}_D$	is the total volume of demand demanded in the simulation.
k_{b}	is the supply output category, see Equation 35.
$\boldsymbol{i}_t$	is the list of all timesteps.
$D_{3D_{\mathrm{D}}}$	is the volume of demand from a consumer group, Equation 109.
$\boldsymbol{j}_{3D_D}$	is the list of consumers by supply, Equation 55.
$D_{2S_{\mathrm{D}}}$	is the demand available (to be bought) from the retailer.
$\boldsymbol{j}_{2S_D}$	is the list of retailers by consumer, Equation 57.
$D_{1S_{\mathrm{D}}}$	is the demand available (to be bought) from the producer.

$\boldsymbol{j}_{1S_D}$ is the list of producers by retailers, Equation 59.

4.7.4 NEW DEMAND

We can show the new demand for the goods D_{nd} as a function largely determined by the volume of new demand at a previous timestep. The volume of new demand D_{nd} at each timestep, varies between economic regions, between business category, and between household subgroups, therefore,

$$D_{nd(\boldsymbol{i}_D,i_t)} = D_{nd(\boldsymbol{i}_D,k_t)} + \delta_{D_{nd}(\boldsymbol{i}_D,i_t)}, \quad 112$$

where:

$\boldsymbol{i}_D$ is the list of consumers, see Equation 36 (Section 4.1.1.2.1),
i_t is the simulation timestep,
k_t is the previous simulation timestep,
$\delta_{D_{nd}}$ is the increment of variation in the demand of supplies.

4.8 *RESOLVING SALES* (*STEP 7*)

The demand demanded was updated at the start of these calculations, based on relative price stability (Step 5, Section 4.6) and consumer behaviour (Step 6, Section 4.7). The feedback to the retailer of stock selling out or stock remaining unsold is calculated in this section (Step 7).

For simplicity of a first approach, let's assume that the supply chain can distribute the available supplies from the producer via the retailers to the consumer. In short, if the retailers' shelves run low of stock, they order more sock and if it is available, it will be delivered. The limitation of this assumption is that it does not differentiate between the efficiency of different supply chains or the purchasing power of retailers to bias the availability of supplies through their stores. Similarly, it does not differentiate between the buying power of consumers. For example, if a bakery cannot produce enough bread to meet demand, say, due to a shortage of flour, they may prioritise delivering bread and cakes to business customers, such as restaurant and cafes, before sell directly to non-business customers. To implement competition between retailers or between consumers becomes significantly more mathematically complex and vastly more computationally intensive.

4.8.1 SUPPLY DEMANDED (SOLD)

The supplies demanded represents the volume of supplies sold. If demand is less than supplies, then some supplies will remain unsold and available for purchase in the next timestep after accounting for sell by date spoilage.

The supply level must be resolved, based on the sum of the supplies available from a given producer against the summed demand from each consumer group for that supplier, via the selected retailer. The process starts by determining the ratio of the demand to the supplies available.

$$\sigma_{S_{1S_D}(k_{1S_S},i_t)} = \frac{D_{3D_D(j_{3D_S},i_t)}}{S_{1S_S(k_{1S_S},i_t)}}, \quad 113$$

where:

k_{1S_S} is the specific business group as a producer (supply side), see Equation 51.
i_t is the current timestep of output supply production.
D_{3D_D} is the volume of demand from a consumer group, Equation 109.

$\boldsymbol{j}_{3D_S}$	is the list of consumers by supplier, Equation 53.
S_{1S_S}	is the volume of supplies by retailer for each consumer group.

Note that the indexing of demand is based on the supply index relationship and not the demand indexing relationship, which is necessary to convert the discretisation of demand demanded into the form of supplies supplied.

If the ratio of demand to supplies is greater than one, then some demand will be unfulfilled, so the ratio must be constrained by an upper limit of unity,

$$\sigma_{S_{1S_D}}\left(k_{1S_S}, \boldsymbol{i}_t\right) > 1\,,\ \sigma_{S_{1S_D}}\left(k_{1S_S}, \boldsymbol{i}_t\right) = 1\,. \qquad 114$$

Then the supplies demanded, i.e., sold, is the constrained ratio of demand to supplies, as a factor or the available supplies,

$$S_{1S_D}\left(k_{1S_S}, \boldsymbol{i}_t\right) = \sigma_{S_{1S_D}}\left(k_{1S_S}, \boldsymbol{i}_t\right) S_{1S_S}\left(k_{1S_S}, \boldsymbol{i}_t\right)\,. \qquad 115$$

The supplies that are unsold can be resolved as follows,

$$\Delta S_{1S_S}\left(k_{1S_S}, \boldsymbol{i}_t\right) = S_{1S_S}\left(k_{1S_S}, \boldsymbol{i}_t\right) - S_{3D_D}\left(k_{1S_S}, \boldsymbol{i}_t\right)\,. \qquad 116$$

4.8.2 DEMAND SUPPLIED (PURCHASED)

The demand supplied represents the volume of supplies purchased. If demand is greater than supplies, then the supplies will sell out and the remaining demand will be unfulfilled. The demand must be resolved: the demand available from a given consumer group against the summed supplies from each producer group for that supply, via the selected retailers used by subgroups within the consumer group. The process starts by determining the ratio of the supplies to the demand available to that producer group.

$$\sigma_{D_{3D_S}}\left(k_{3D_D}, \boldsymbol{i}_t\right) = \frac{S_{1S_D}\left(j_{1S_D}, \boldsymbol{i}_t\right)}{D_{3D_D}\left(k_{3D_D}, \boldsymbol{i}_t\right)}\,, \qquad 117$$

where:

k_{3D_D}	is the consumer group (demand), see Equation 55.
i_t	is the current timestep of output supply production.
S_{1S_D}	is the volume of supplies demanded from the producer group.
$\boldsymbol{j}_{1S_D}$	is the list of producers by retailers, Equation 59.
D_{3D_D}	is the volume of demand from a consumer group, Equation 109.

Note that the indexing of supplies is based on the demand index relationship and not the supply indexing relationship, which is necessary to convert the discretisation of supplies supplied into the form of demand demanded.

If the ratio of demand to supplies is less than one, the supplies will sell out and some demand will be unfulfilled, so the ratio must be constrained by a lower limit of unity,

$$\sigma_{D_{3D_S}}\left(k_{3D_D}, \boldsymbol{i}_t\right) > 1\,,\ \sigma_{D_{3D_S}}\left(k_{3D_D}, \boldsymbol{i}_t\right) = 1\,. \qquad 118$$

Then the demand supplied, i.e., purchased, is the constrained ratio of supplies to demand, as a denominator of the available demand,

$$D_{3D_S(k_{3D_D},\boldsymbol{i}_t)} = \frac{D_{3D_D(k_{3D_D},\boldsymbol{i}_t)}}{\sigma_{D_{3D_S}(k_{3D_D},\boldsymbol{i}_t)}}. \quad 119$$

The demand that is unfulfilled can be resolved as follows,

$$\Delta D_{3D_S(k_{3D_D},\boldsymbol{i}_t)} = D_{3D_D(k_{3D_D},\boldsymbol{i}_t)} - D_{3D_S(k_{3D_D},\boldsymbol{i}_t)}. \quad 120$$

4.8.3 RELATIONSHIP BETWEEN THE LEVELS OF DEMAND SUPPLIED

The relationship between the demand supplied in each of the demand object vectors is represented as,

$$\overline{\overline{D}}_{S(k_b,\boldsymbol{i}_t)} = D_{3D_S(\boldsymbol{j}_{3D_D},\boldsymbol{i}_t)} = D_{2S_S(\boldsymbol{j}_{2S_D},\boldsymbol{i}_t)} = D_{1S_S(\boldsymbol{j}_{1S_D},\boldsymbol{i}_t)}, \quad 121$$

where:

$\overline{\overline{D}}_S$	is the volume of demand supplied (supplies purchased) in the simulation.
k_b	is the supply output.
i_t	is the current timestep of output supply production.
D_{1S_S}	is the volume of demand supplied, purchased from a producer.
$\boldsymbol{j}_{3D_D}$	is the list of consumers by supply, Equation 55.
D_{2S_S}	is the volume of demand supplied, purchased from a retailer.
$\boldsymbol{j}_{2S_D}$	is the list of retailers by consumer, Equation 57.
D_{3D_S}	is the volume of demand supplied, purchased by a consumer group.
$\boldsymbol{j}_{1S_D}$	is the list of producers by retailer by consumer, Equation 59.

4.9 *BUSINESS BEHAVIOUR* (*STEP 8*)

4.9.1 SUPPLIES SUPPLIED

Supplies supplied is the volume of supplies available for sale to customers within the current timestep. The business approach to supply volume decision-making is discussed in Section 2.1.3.4. The options are:

- Elastic increase or decrease in production.
- Non-elastic increase or decrease in production.
- Relocate the business production to a different economic region.
- Existing businesses cease trading.
- New businesses start trading.

This business behaviour, in response to supplies remaining unsold or selling out can be represented as surrogate modelling function $f_{S_{1S}}(\quad)$. Within the brackets of the function are the list of indices that define the variables and dimensions for the pattern fitting model. The model parameter vector $\boldsymbol{w}$ is determined by a training process against the population data. Once trained, the surrogate model will provide an *interpolated* predicted response for each unique set of input index values by interrogating the value surrogate model. The output from the function will allocate the change in production behaviour or price setting.

The surrogate model must output a revised supply object (see Equation 67 and Equation 72), using the index relationship of the supplier and producer by the consumer $\boldsymbol{j}_{[1S,2S]_{b_D}}$. The list of producers

$\boldsymbol{j}_{1S_S}$ producing a supply $\boldsymbol{i}_b$, may change if new businesses start trading and existing businesses cease trading, which will be detailed in Papers 04 and 05 (Maybury, in progress) respectively. The total number of producer categories N_{1S_S} must be updated. The vector $\boldsymbol{A}_{1S_S}$ (Equation 69, above) will need to have the value of new supplies S_{ns} updated to reflect any business decision on elastic production or any non-elastic changes following the appropriate lead time following the decision.

The vector of the retailers $\boldsymbol{j}_{2S_S}$ may change if the producers change the distributor or the supplies. The total number of retailer categories N_{2S_S} must be updated. The vector $\boldsymbol{A}_{2S_S}$ (Equation 70) must have its values updated for volume of supplies through the retailer S_{2S_S}, and the ratio of demand from the consumer category σ_{2S_S}.

The ratio of supplies from a producer group distributed through a given retailer group is,

$$\sigma_{2S_S(j_{2S_S},i_t)} = \frac{S_{2S_S(j_{2S_S},i_t)}}{S_{1S(i_b,i_t)}}, \qquad 122$$

where:

$\boldsymbol{j}_{2S_S}$	is the list of retailers by producer, Equation 52.
$\boldsymbol{i}_b$	is a list of supply categories, Equation 35.
$\boldsymbol{i}_t$	is the list of all timesteps.
S_{1S_S}	is the volume of supplies from a producer group, Equation 126.
S_{2S_S}	is the volume of supplies distributed through a retailer group.

The new supplies S_{nd} can be represented as a function determined by the volume of new supplies at a previous timestep. The volume of new supplies S_{nd} at each timestep, varies between economic regions, between business category, and between household subgroups, therefore,

$$S_{ns(\boldsymbol{i}_{1S},\boldsymbol{i}_t)} = S_{ns(\boldsymbol{i}_{1S},k_t)} + \delta_{S_{ns}(\boldsymbol{i}_{1S},\boldsymbol{i}_t)}\,, \qquad 123$$

where:

$\boldsymbol{i}_{1S}$	is the list of producers and manufacturers, see Equation 45 (Section 4.1.1.2),
i_t	is the simulation timestep,
k_t	is the previous simulation timestep,
$\delta_{S_{ns}}$	is the increment of variation in the production of new supplies.

The vector of the consumers $\boldsymbol{j}_{3D_S}$ may change based on consumer behaviour (Section 4.7). The total number of consumer categories N_{3D_S}must be updated. The vector $\boldsymbol{A}_{3D_S}$ (Equation 71) must have its values updated for the volume of supplies demanded by the consumer S_{3D_D}.

4.9.2 Surrogate Model

The updates to the demand object $o_{S(i_t)}$ by the surrogate function $f_{S_{1S}}(\quad)$ is summarised in Equation 124, below.

$$o_{S(i_t)}\left[N_{1S_S}, \boldsymbol{j}_{1S_S}, N_{2S_S}, S_{ns}, \boldsymbol{j}_{2S_S}, N_{3D_S}, S_{2S_S}, \sigma_{2S_S}, \boldsymbol{j}_{3D_S}, S_{3D_D}\right] = f_{S_{1S}}\left(\boldsymbol{j}_{[3D,2S]_{b_{1S}}}\right), \qquad 124$$

where:

i_t	is the current timestep of output supply production.

N_{1S_S}	is the total number of producer categories; length of vector $\boldsymbol{j}_{1S_S}$.
$\boldsymbol{j}_{1S_S}$	is the list of producers by supply, Equation 51.
N_{2S_S}	is the total number of retailer categories; length of vector $\boldsymbol{j}_{2S_S}$.
S_{ns}	is the newly created supplies in the timestep.
$\boldsymbol{j}_{2S_S}$	is the list of retailers by producer, Equation 52.
N_{3D_S}	is the total number of consumer categories; length of vector $\boldsymbol{j}_{3D_S}$.
S_{2S_S}	is the supply available to be sold by the retailer.
σ_{2S_S}	is the ratio of supplies supplied from the retailer from the producer.
$\boldsymbol{j}_{3D_S}$	is the list of consumers by retailer by producer, Equation 53.
S_{3D_D}	is the supply demanded by the consumer.
$\boldsymbol{j}_{[3D,2S]_{b_{1S}}}$	is the list of consumers and the retailers by producer, Equation 65.
k_t	is the previous timestep of output supply production.

The matter of business viability requires the knowledge of the business turnover and running costs, so viability, and non-elastic decisions must be handled by the financial forcing function for businesses ([Paper 05], in progress). The creation of new entities will be handled by Paper 04, Consumer Life Cycles (in progress).

4.9.3 Relationship between the levels

The relationship between the supply variables in each of the supply vectors in the supply object is represented as,

$$\bar{\bar{S}}_{S(k_b,\boldsymbol{i}_t)} = S_{1S_S(j_{1S_S},\boldsymbol{i}_t)} = S_{2S_S(j_{2S_D},\boldsymbol{i}_t)} = S_{3D_S(j_{3D_S},\boldsymbol{i}_t)}\,, \qquad 125$$

where:

$\bar{\bar{S}}_S$	is the total volume of supplies supplied in the simulation.
k_b	is the supply categories, Equation 35.
$\boldsymbol{i}_t$	is the list of all timesteps.
$\boldsymbol{j}_{1S_S}$	is the list of producers by supply, Equation 51.
S_{1S_S}	is the volume of supplies supplied from a producer group, Equation 126.
$\boldsymbol{j}_{2S_S}$	is the list of retailers by producer, Equation 52.
S_{2S_S}	is the supply available to be sold by the retailer.
$\boldsymbol{j}_{3D_S}$	is the list of consumers by retailers, Equation 53.
S_{3D_S}	is the supply available to be sold to the consumer.

4.9.4 New supply

The computational process only needs to address the change in behaviour of businesses. This change could be caused by disruption within the market, such as drought or war affecting the price of input supplies $Q_{\Sigma ds_\beta}$. To represent this change, we define the volume of newly produced supplies S_{ns} by using the baseline volume from the previous simulation period and then add an increment on that baseline. The volume of new supplies S_{ns} at each timestep i_t, varies between economic regions, and business category, such that,

$$S_{1S_S(\boldsymbol{i}_{1S},i_t)} = S_{ns(\boldsymbol{i}_{1S},i_t)} + \sigma_{S_{\Delta s}(\boldsymbol{i}_{1S},k_t)} \Delta S_{1S_S(\boldsymbol{i}_{1S},k_t)}\,, \qquad 126$$

where:

$\boldsymbol{i}_{1S}$ is the list of producers, see Equation 45 (Section 4.1.1.2).
i_t is the simulation timestep.
k_t is the previous simulation timestep.
$\sigma_{S_{\Delta s}}$ is the increment of unsold supplies surviving to the next timestep.
ΔS_{1S_S} is the unsold supply from the previous timestep, Equation 116.

5 DISCUSSION

The mathematical framework presented above introduces the first testable theory of the pricing of supplies. This section defines the limitations of the model and explores the broader implications of using validated models for monetary and fiscal policies.

5.1 A NEW THEORY OF PRICING

Current theories of supply and demand cannot be validated against observed data. They claim that the price of supplies is determined by the relationship between the total supply and total demand at an economy level. Yet, they cannot explain how retailers set the price without knowledge of real-time supply and demand.

The new theory presented here is based upon the following observations:

- Businesses set the retail price.
- Business input costs have regional and industrial sector variation.
- Demand is inherently stable for essential supplies, such as food and drink.
- Retail price affects consumer choice.
- Businesses have incomplete information about demand.

A model has been proposed that defines the likely decision-making processes of retailers within the Economy Dynamics large-scale economic forecasting framework. The model is presented in two formulations: the first represents the observable processes of businesses and should align with the experience of people involved in these processes, and the second transforms this into the Economy Dynamics computational framework.

In addition, by modelling the cost price of production, the theory can calculate the non-monetary price too, such as the volume of greenhouse gases generated by the manufacture and distribution of supplies.

5.2 THE UGLY SOLUTION

When developing a simulation framework of a complex system within engineering, it is not realistic to expect an author to define an analytical framework from scratch within a paper without having built the software and debugged the mathematics and conceptual framework architecture first. For example, the structure of the dataset, its relationship to other datasets and the sequence of data manipulations by the analytical models are easier to resolve within the computer code rather than by thinking through the analytical scenarios.

The phrase often used when defining a set of complex algorithms is to "write an ugly solution and then refine it". This means, once a 'simple' solution is defined, it can be improved within the software code. In contrast, seeking to define a complete solution is often impractical, and if possible, highly time consuming. Suffice to say, this paper represents the first draft of the approach to modelling the price of supplies, without being implemented in a software simulation environment. The list of equations may not be complete, there are undoubtably errors in the equations, and the use of variables and indexing must be verified.

The equations in this paper have been selected as a simple first approach. For example, the methods to determine the retail price based on business competition (Section 4.4) only consider three business types: those that compete on price alone, those that have customer loyalty accepting a price change tolerance above other suppliers, and those that are independent of the pricing of competitors. If the model cannot respond to sharply rising prices like those following the February 2022 oil and gas price surge, then new and more complex models may need to be developed.

The surrogate models for estimating the minimum mean profit (Section 4.5), consumer behaviour (Section 4.7.2), and producer decision-making (Section 4.9.2) are large research topics. This paper shows the place holder for a method, which must be developed by training an appropriately selected algorithm against historical datasets. The equations presented here (Equations 103, 110 & 124) define the required output variable(s) and the likely input variables into the surrogate model.

5.3 VALIDATING THE PRICING MODEL

Within the completed Economy Dynamics framework, features of an economy such as inflation can be calculated from the simulation results of the whole economy, i.e., any metric that can be measured in an economy can be measured from the simulations. The validation metrics that can be calculated include Gross Domestic Product *GDP*, the proportion of households whose income falls below their cost-of-living (Maybury, 2020 [Paper 01]), and the rate at which businesses fail (Maybury, in progress [Paper 05]). The validation will initially be for trends in behaviour, such as predicting the rate of change in GDP, as absolute predictions, i.e., the specific value of GDP, will require substantial tuning of the Economy Dynamics models.

The approach to validation will follow the principles of validating climate change forecasting models, such as the approach implemented by Hausfather *et al*. (2019). This means that the forecast is to be generated by inputting the actual parameter values from historical data. The output of the model can then be compared to the validation variables of that historical data. For example, in forecasting climate change, validation of a model requires the actual values of CO_2 produced by human activity to be used as inputs. Then the model can predict the measured effect on global temperatures, which are compared to the observed past temperatures to establish how well the predictions match the real-world data. The same approach can be applied to the forecasting of price, whereby environmental and political effects on business input supplies can be implemented from the real events and then inflation can be predicted and compared to historical data.

A significant limitation to this validation process is the availability of suitable datasets to train the surrogate models and to validate the regional forecasts. Currently, existing data must be disaggregated by economic region, and by business and by household economic subgroups. This is a substantial task. However, given financial support for the project then appropriate data can be collated. For now, this paper demonstrates the concepts involved in determining the price of supplies within a model that can be validated.

5.4 *WHY IT IS ESSENTIAL TO VALIDATE THEORIES*

The use of unvalidated economic theories have far reaching consequences when applied to economic policy. Economic policy consists of fiscal policy, determining the collecting and distribution of tax revenues, and monetary policy, to manage the value of currency. Since economic policies invariably disadvantage some groups of households and businesses, compared to others, they are political decisions. Estimating the immediate effect of a policy on household and business finances, such as a new government budget, requires a form of accountancy as it is applied to the current snapshot of the economy and does not involve forecasting. In the UK, this role is performed by the Office for Budgetary Responsibility (OBR). When a new budget is announced by a UK Chancellor the OBR establishes how this will affect the financial circumstances of various groups of people and businesses in the next year.

However, economic policy will have a medium- and long-term impact that will affect businesses, and therefore households, in different ways. Moreover, it can have non-financial impact too, such as influencing the ethics and sustainability of business practices. Understanding the future impact of economic policies on an economy is not accountancy, but economics. Currently, assessing the economic effects of policies is political opinion. As economic theories cannot be validated, policymakers will be influenced by their views of social hierarchy and ethics, in their belief of what is best for the economy. Furthermore, whether their policy achieves their objectives is not currently testable.

In the subsections below, the benefits of basing policymaking on validated economic theories is discussed for monetary and fiscal policy.

5.4.1 MONETARY POLICY

In 2020, in response to the economic impact of the Corona Virus pandemic, the world's central banks with responsibility for managing the value of the U.S. Dollar, Euro, Japanese Yen and Great British Pound, created new money worth $8 trillion. This action is based on an economic policy termed *quantitative easing*. It is intended to alleviate a lack of economic growth believed to be caused by insufficient money in the hands of consumers. This policy is underpinned by the unvalidated concepts of *quantity theory* and *the special nature of the markets* (Maybury, 2022 [Essay 02]).

As a result of these unvalidated theories, the trigger for central banks generating new money and the choice of where to spend it is based on unjustifiable opinion. The central banks created vast sums of money to buy back government debt from investors, in the form of gilts and bonds. Yet, they can neither evaluate, nor predict, the potential adverse effects of this action. For example, how much of this money subsequently ends up invested in property? What effect does this have on house prices and rents during the following years? How does this affect inflation and the cost of living in different regions of the economy? The central banks do not know this information, which is fundamental to their role in managing the value of currency. They cannot demonstrate that there was any proportionate benefit to their actions.

In contrast, by validating economic theories, the central banks could establish whether the current tools of monetary policy are beneficial or detrimental to managing the value of currency. Moreover, given the effects of the financial and commodity markets on the value of currency, surely understanding whether they should remain centralised, acting like a monopoly, or distributed, with several competing independent markets, is vital. Similarly, understanding whether automated highfrequency financial trading and short-selling destabilise an economy is essential. However, this requires policymakers to use validated tools to provably demonstrate how global events and the response of current economic structures affect the cost-of-living and how monetary (and fiscal) policies can address this.

5.4.2 Fiscal Policy

The measure of the size of an economy, as defined by the calculation of GDP, is the wealth generated by businesses. All taxes collected originate from the wealth generated by businesses, wherever it is taken from, be it directly from businesses or indirectly from the owners and employees through income and so-called consumer taxes. The current approach to tax collection is historical and political and has not been created to suit a modern economy. The issues caused by fiscal policy and the feasibility of a business focused approach are discussed by Maybury, 2023 [Essay 03]. To understand the effectiveness of the approach and its future effects, in terms of maximising revenues without any adverse effects on job creation, requires a validated forecasting framework.

5.5 Further work

Validating the price of supplies model requires the Economy Dynamics forecasting software and data structures to be developed. The complete framework requires six more models to be published before the development can be undertaken. These models are:

- **Paper 04.** Consumer life cycles: the dynamic change of households and businesses. A model of the probability of household and business life cycle events occurring, such as birth and death, start-up and bankruptcy.
- **Paper 05.** Modelling decision-making from population data: Business Income Forcing. The probability of businesses making financial decisions under a given set of economic circumstances can be captured from population data by using surrogate modelling methods.
- **Paper 06.** The price of land. Evaluating the value of land (and assets) to the different economic groups of households and businesses to determine the unit price set by successful buyers.
- **Paper 07.** The wage rate for labour. Evaluating the value of employees to businesses, between regions of an economy and between industrial (and public) sectors.
- **Paper 08.** Validation of the Economy Dynamics analytical framework. An approach to automating the processes of comparing retrospective forecasts to observed characteristics of an economy through a software testing pipeline for source code acceptance.
- **Paper 09.** Computational framework for large-scale economic forecasting. Defining the computational framework and the simulation process for an applied mathematics approach to forecasting economy behaviour.

Once the sub-models have been presented in conceptual form, the documentation for code development will be prepared to enable a collaborative approach to the software development. Then when the prototype forecasting software is complete, and the surrogate models have been derived, validation of the models will be performed.

5.6 CONCLUSION

This paper proposes a new testable theory of pricing to address the limitations of existing supply-and-demand models, which fail to explain how retailers set prices in practice. The new theory centres on modelling the decision-making processes of retailers, incorporating factors like input costs, stable demand for essentials supplies, incomplete real-time information about supply and demand, and business competition.

The need for validating economic theories has been emphasised by highlighting the potentially disastrous consequences of using unvalidated theories to inform monetary and fiscal policies. The validation process not only ensures accuracy but also allows for comparison, enabling us to identify the best current explanation. Moreover, as engineering demonstrates, complexity can be built up progressively and by teams of independent researchers. This is demonstrably useful for adapting and controlling the world we live in.

Modelling retailer decision-making in setting the price of supplies is complex. The methodology presented is a simplified model requiring further refinement through software implementation. However, the framework offers a new lens for studying economic behaviour, as it can be validated. Therefore, it lays the groundwork for progressively building knowledge of economic systems, allowing theories to be developed and validated independently, thereby fostering collaboration and advancing the field.

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This is a substantial undertaking, currently being performed in the author's spare time, without funding for the support of economists, software developers and engineers. With suitable funding the development and validation of the Economy Dynamics framework could be completed within two years. As it currently stands, writing this paper took three years duration, but is equivalent to just 10 weeks of continuous effort.

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